

E. L. Hammon

THE
NEW COMPLETE SYSTEM
OF

ARITHMETIC,

COMPOSED FOR THE
USE of the CITIZENS of the UNITED STATES.

By **NICOLAS PIKE, A. M.**
MEMBER of the AMERICAN ACADEMY of ARTS and SCIENCES.

ABRIDGED
FOR THE
USE OF SCHOOLS.

THE THIRD EDITION, CORRECTED AND ENLARGED.

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1798.

RECOMMENDATIONS

of this Work at large.

Dartmouth University, A. D. 1786.

AT the request of Nicolas Pike, Esq. we have inspected his system of Arithmetic, which we cheerfully recommend to the public as easy, accurate and complete. And we apprehend there is no treatise of the kind extant, from which so great utility may arise to Schools.

E. WOODWARD, Math. and Phil. Prof.

JOHN SMITH, Professor of the Learned Languages.

I do most sincerely concur in the preceding recommendation.

J. WHEELLOCK, President of the University.

Providence, State of Rhode Island, 1785.

WHOEVER may have the perusal of this treatise on Arithmetic may naturally conclude I might have feared myself the trouble of giving it this recommendation, as the work will speak more for itself than the most elaborate recommendation from my pen can speak for it; but as I have always been much delighted with the contemplation of mathematical subjects, and at the same time fully sensible of the utility of a work of this nature, was willing to render every assistance in my power to bring it to the public view: And should the student read it with the same pleasure with which I perused the sheets before they went to the Press, I am persuaded he will not fail of reaping that benefit from it, which he may expect, or wish for, to satisfy his curiosity in a subject of this nature. The Author, in treating on numbers, has done it with so much perspicuity and singular address, that I am convinced the study thereof will become more a pleasure than a task.

The arrangement of the work, and the method by which he leads the Tyro into the first principles of numbers, are novelties I have not met with in any book I have seen. Wingate, Hatton, Ward, Hill, and many other Authors, whose names might be adduced, if necessary, have claimed a considerable share of merit, but when brought into a comparative point of view with this treatise, they are inadequate and defective. This volume contains, besides what is useful and necessary in the common affairs of life, a great fund for amusement and entertainment. The mechanic will find in it much more than he may have occasion for; the Lawyer, Merchant and Mathematician, will find an ample field for the exercise of their genius; and I am well assured it may be read to great advantage by students of every class, from the lowest school, to the University. More than this need not be said by me, and to have said less, would be keeping back a tribute justly due to the merit of this work.

BENJAMIN WEST.

University in Cambridge, A. D. 1786.

HAVING, by the desire of Nicolas Pike, Esq. inspected the following volume in manuscript, we beg leave to acquaint the Public, that in our opinion it is a work well executed, and contains a complete system of Arithmetic. The rules are plain, and the demonstrations perspicuous and satisfactory; and we esteem it the best calculated of any single piece we have met with, to lead youth, by natural and easy gradations, into a methodical and thorough acquaintance with the science of figures. Persons of all descriptions may find in it every thing, respecting numbers, necessary to their business; and not only so, but if they have a speculative turn and mathematical taste, may meet with much for their entertainment at a leisure hour.

We are happy to see so useful an American production, which, if it should meet with the encouragement it deserves, among the inhabitants of the United States, will save much money in the country, which would otherwise be sent to Europe, for publications of this kind.

We heartily recommend it to schools, and to the community at large, and with that the industry and skill of the Author may be rewarded, for so beneficial a work, by meeting with the general approbation and encouragement of the Public.

JOSEPH WILLARD, D. D. President of the University.

E. WIGGLESWORTH, S. T. P. Hollis.

S. WILLIAMS, L. L. D. Math. et. Phil. Nat. Prof. Hollis.

Tale College, 1786.

UPON examining Mr. Pike's System of Arithmetic and Geometry in Manuscript, I find it to be a Work of such Mathematical Ingenuity, that I esteem myself honored in joining to the Reverend President Willard, and other learned Gentlemen, in recommending it to the Public as a production of Genius, interspersed with Originality in this Part of Learning, and as a Book suitable to be taught in Schools—of Utility to the Merchant, and well adapted even for the University Instruction. I consider it of such Merit, as that it will probably gain a very general Reception and Use throughout the Republic of Letters.

EZRA STILES, President.



PREFACE to the FIRST EDITION.

AN abridgment of PIRK's universally approved Arithmetic having been earnestly solicited from various parts of the United States, the present proprietor of the work at large, desirous to gratify the public, has with pleasure complied with the general wish.

This celebrated work, which is now used as a classical book in all the Newengland Universities, is, by all who have examined it, allowed to excel every thing of the kind on this continent: He has reason to hope this Abridgment will not be less esteemed as a school book, as the author has done it in such a manner as to exclude nothing material in the arithmetical part.

This work is calculated to convey instruction in as easy, concise and familiar a manner, as the nature of the subject will admit, and therefore peculiarly adapted to the use of schools: and were every pupil possessed of a book, the Preceptor would be able to instruct more than double the number than otherwise could be taught: As the time taken up in setting sums and writing the rules might be much more profitably spent in explaining them to the various capacities of the children.

There are two capital errors, into which authors on this subject have generally fallen.

The one consists in not exemplifying the rules by a particular application of them in examples, wrought at large: for although it may be thought that this method will stimulate children to greater exertions, in practical experience it will be found quite the reverse, and that the greater part will sink into absolute despondency, unless they have some clue besides a meer rule.

The other is the opposite extreme, and is, if possible, more faulty than the former; viz. when all the questions are wrought at large. In this case, the pupil is entirely prevented from exercising his reasoning faculties,

in attending to the nature and reason of the operations, and insensibly acquires an inert habit of mind, and, being unaccustomed to a habit of reasoning, is prepared to receive any proposition whatever upon the bare ipse dixit of another.

But this is not the only ill consequence of this error : For the book in which all the questions are wrought at large, if designed for a proper system, must necessarily be so bulky as by far to exceed the price of a common school book ; otherwise, it can contain but a meer smattering of Arithmetic.

The author has happily executed this Abridgment on a plan, which is an exact medium between the two extremes before mentioned, the rules are sufficiently elucidated by a particular application of them to practice, in one or two examples at large, as the nature of the several rules required ; and the other questions are given with their answers for the exercise of the pupil ; and the sale of nearly five thousand of the original work, in a short time, is the strongest confirmation that this method is well approved of by an enlightened public.

If it would add to the merit of this work to mention that other authors have made selections from PIRK'S Arithmetic, to enhance the value of their treatises on this subject, we could notice a late author who has made copious extracts from it to enrich his own work with : But as this circumstance is notorious, the time of the reader need not be taken up with the recital.

The proprietor doubts not but the merit of this work at large will continue to be noticed by a discerning public ; and that this Abridgment will receive the encouragement which it may deserve.

With the greatest respect, the public's most obedient and very humble servant,

ISAIAH THOMAS.

WORCESTER, April 30, 1793.

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— Signifies the second power or Square	—
— Signifies the third power or Cube	—

EXPLANATION of CHARACTERS used in this TREATISE.

$=$ THE sign of Equality : As 12 Pence = 1 Shilling, signifies that 12 Pence are equal to 1 Shilling; and, in general, that whatever precedes it is equal to what follows.

$+$ The sign of Addition : As, $5 + 5 = 10$, that is, 5 added to 5 is equal to 10. — Read 5 plus 5, or 5 more 5 equal to 10.

$-$ The sign of Subtraction : As, $12 - 4 = 8$, that is, 12 lessened by 4 is equal to 8, or 4 from 12 and 8 remains. — Read 12 minus 4, or 12 less 4 equal to 8.

$\times$ The sign of Multiplication : As, $6 \times 5 = 30$, that is, 6 multiply by 5 is equal to 30. — Read 6 into 5 equal to 30.

$\div$ or $6)30$ The sign of Division : As, $30 \div 5 = 6$, that is, 30 divided by 5 is equal to 6. — Read 30 by 5 equal to 6.

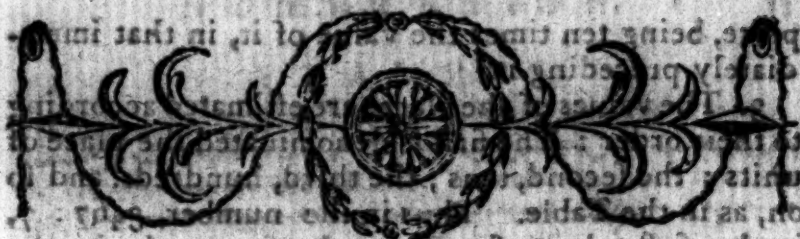
$\frac{875}{25}$ Numbers, placed fractionwise, do likewise denote division, the Numerator or upper number being the dividend, and the Denominator or lower number, the divisor, thus $\frac{875}{25}$ is the same as $875 \div 25 = 35$.

$8 : 16 :: 2 : 4$ The sign of Proportion, thus, $8 : 16 :: 2 : 4$: Shews that the difference between 8 and 9 added to 6 is equal to 13. — Read 9 minus 2 plus 6 equal to 13; and that the line a top (called a Vinculum) connects all the numbers over which it is drawn.

$\overline{9-2+6=13}$ Signifies that the sum of 3 and 5 taken from 12 leaves or is equal to 4.

$\overline{-^2}$ Signifies the second Power or Square.

$\overline{-^3}$ Signifies the third power or Cube.



in the first place, figures only seven; 0, in the second place, against 0 read or say; 1, in the third place, four hundred 2, in the fourth place, three thousand, and the whole, taken together, is read thus; three thousand four hundred and fifty seven. **P I K E**

ARITHMETIC.

is the Art or Science of computing by numbers, and is comprised under five principal or fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION. **A B R I D G E D.**

ARITHMETIC is the Art or Science of computing by numbers, and is comprised under five principal or fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

NUMERATION.

Teaches the different value of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the rest are called figures or digits.

2. Besides the simple value of figures, as above noted, they have, each, a local value, according to the following law; viz. In a combination of figures, reckoning from right to left, the figure, in the first place, represents its primitive simple value; that in the second place, ten times its simple value, and so on; the value of the figure, in each succeeding place,

place, being ten times the value of it, in that immediately preceding it.

3. The values of the places are estimated according to their order : The first is denominated the place of units ; the second, tens ; the third, hundreds, and so on, as in the Table. Thus in the number, 3467 : 7, in the first place, signifies only seven ; 6, in the second place, signifies 6 tens, or sixty ; 4, in the third place, four hundred ; 3, in the fourth place, three thousand ; and the whole, taken together, is read thus ; three thousand four hundred and sixty seven.

4. A cypher, though it is of no signification itself, yet, it possesses a place, and, when set on the right hand of figures, in whole numbers, increases their value in the same tenfold proportion ; thus, 9 signifies only nine ; but, if a cypher is placed on its right hand, thus, 90, it then becomes ninety.

To enumerate any parcel of figures, observe the following Rule.

First, commit the words, at the head of the Table, viz. units, tens, hundreds, &c. to memory ; then, to the simple value of each figure, join the name of its place, beginning at the left hand, and reading towards the right.—*More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands.—2. Place the figures 1, 2, 3, 4, &c. as indices, over the 2d, 3d, 4th, &c. period : These indices will then shew the number of times the millions are involved—the figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions (or millions of millions of millions.)

EXAMPLES.

NUMERATION.

EXAMPLE.

Sextilli.	Quintill.	Quatrill.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
913,208,000,	341,620,	067,219,	356,809,	379,120,	406,129,	763
Thousands	Thousands	Thousands	Thousands	Thousands	Thousands	

NOTE 1. Billions is substituted for millions of millions : Trillions, for millions of millions of millions : Quatrillions, for millions of millions of millions of millions.

Quintillions, Sextillions, Septillions, Octillions, Nonmillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote.

NOTE 2. The right hand figure of each half period has the place of units of that half period ; the middle one, that of tens, and the left hand one, that of hundreds.

The APPLICATION.

Write down, in proper figures, the following numbers.

Fifteen.

Two hundred and seventy nine.

Three thousand, four hundred and three.

Thirty seven thousand, five hundred and sixty seven.

Four hundred, one thousand and twenty eight.

Nine millions, seventy two thousand and two hundred.

Fifty five millions, three hundred, nine thousand and nine.

Eight hundred millions, forty four thousand, and fifty five.

Two thousand, five hundred and forty three millions, four hundred and thirty one thousand,

seven hundred and two.

Write

12 SIMPLE ADDITION.

Write down, in words at length, the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49168189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One. II. Two. III. Three. IV. Four. V. Five. VI. Six. VII. Seven. VIII. Eight. IX. Nine. X. Ten. XI. Eleven. XII. Twelve. XIII. Thirteen. XIV. Fourteen. XV. Fifteen. XVI. Sixteen. XVII. Seventeen. XVIII. Eighteen. XIX. Nineteen. XX. Twenty. XXX. Thirty.	XL. Forty. L. Fifty. LX. Sixty. LXX. Seventy. LXXX. Eighty. XC. Ninety. C. Hundred. CC. Two hundred. CCC. Three hundred. CCCC. Four hundred. D or 10. Five hundred. DC. Six hundred. DCC. Seven hundred. DCCC. Eight hundred. DCECC. Nine hundred. M or 100. One thousand. 1000. Five thousand. 10000. Fifty thousand. 100000. Five hundred thousand. MDCCXCVIII. One thousand seven hundred and ninety eight.
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A less literal number, placed after a greater, always augments the value of the greater; if put before, it diminishes it. Thus, VI is 6; IV is 4; XI is 11; IX is 9; &c.

A D D I T I O N

Is the putting together of two or more numbers, or sums, to make them one total, or whole sum.

S I M P L E A D D I T I O N

Is the adding of several integers or whole numbers together, which are all of one kind, or sort; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or sum total, is 39 pounds.

Rule.

SIMPLE ADDITION.

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RULE.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units: After adding up every figure in that column, consider how many tens are contained in their sum, and, placing the excess under the units, carry so many, as you have tens, to the next column, of tens: Proceed in the same manner through every column, or row, and set down the whole amount of the last row.

PROOF.

Begin at the top of the sum, and reckon the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first. Or, cut off the upper line of figures, and find the amount of the rest; then, if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

ADDITION and SUBTRACTION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column, and the other at top, and

SIMPLE ADDITION.

and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 is 13.

When you would subtract: Find the number to be subtracted in the left hand column, run your eye along to the right hand till you find the number from which it is to be taken, and right over it, atop, you will find the difference—as, 8, taken from 13, leaves 5.

1.	2.	3.	4.	5.	6.
£.	Rs.	Cwt.	Miles.	Yards.	£.
1	12	113	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	789	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	69	67890	567891287
7	23	901	4718	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89093	4321234
5678209	833	654321	5682098
6789098	90	1234567	6543218

SUBTRACTION

SIMPLE SUBTRACTION.

15

SUBTRACTION

Teaches to take a less number from a greater, to find a third, shewing the inequality, excess or difference between the given numbers ; and it is both simple and compound.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

R U L E.

Place the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c. then, drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder ; but if the lower figure be greater than the upper, borrow ten, and subtract the lower figure therefrom : To this difference, add the upper figure, which, being set down, you must add one to the ten's place of the lower line, for that which you borrowed ; and thus proceed through the whole.

P R O P O S I T I O N

In either simple, or compound Subtraction, add the remainder and the less line together, whose sum, if the work be right, will be equal to the greater line :— Or, subtract the remainder from the greater line, and the difference will be equal to the less.

EXAMPLES

EXAMPLES.

	1.	2.	3.	4.	5.	6.
	£.	£.	Miles.	Yards.	Feet.	Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
	—	—	—	—	—	—
Rem.						
	—	—	—	—	—	—
Proof.						
	—	—	—	—	—	—

	7.	8.	9.
	100200300400500600700800900	10000	1000000
	98076054082011023045067089	9999	1
	—	—	—
	—	—	—

MULTIPLICATION

May be accounted the most serviceable Rule in Arithmetic. It teaches how to increase the greater of two numbers given, as often as there are units in the less; performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

It consists of three parts.

1. The Multiplicand, or number given to be multiplied, and, commonly, the largest number.
2. The Multiplier, or number to multiply by, commonly, the least number.
3. The Product is the result of the work, or the answer to the question.

SIMPLE

SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION.

Is the multiplying of any two numbers together, without having regard to their signification; as 7 times 8 is 56, &c.

MULTIPLICATION and DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table, for Multiplication: Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or against your multiplier, along at the right hand, and under your multiplicand, you will find the product, or answer.

To learn it, for Division: Find the divisor in the left hand column, and run your eye along the row to the right hand until you find the dividend: then, directly over the dividend, atop, you will find the quotient, shewing how often the divisor is contained in the dividend.

SIMPLE MULTIPLICATION.

C A S E L.

When the multiplier is not more than 12, always placing the greatest number uppermost, set the multiplier underneath, units under units, &c. and begin as the Table directs, setting down the unit figure under units, and carrying the tens to the next place, in all respects as in simple addition.

P R O O F.

Multiply the multiplier by the multiplicand.

E X A M P L E S.

1.	2.	3.	4.
37934	769308	4980076	763896
2	3	4	5
Prod.			
5.	6.	7.	8.
67589	503764	3918295	9164785
6	7	8	9
9.	10.	11.	
4879567	5864734	8593478649	
10	11	12	

C A S E II.

When the multiplier consists of more places than one, multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the first figure of each product exactly under its multiplier: lastly add these

SIMPLE MULTIPLICATION. 19

products together, in the same order as they stand, and their sum will be their total product.

EXAMPLES.

1.	2.	3.
6357534	8324629	4629845
47	59	76
44502738		
25480136		
Prod. 298804098		
4.	5.	6.
647906	760483	91867584
4873	9152	6875
3157245938	6959940416	631589640000

CASE III.

When the multiplier is a composite number, that is, when it is produced by the multiplication of any two numbers in the Table, multiply the multiplicand by one of those figures, first, and that product by the other : And the last product will be the total required.

EXAMPLES.

1.	2.	3.
Mult. 59375 by 35.	39187 by 48.	91632 by 56.
7		
7x5=35		
415625		
5		
2078125		

Mult.

20 SIMPLE MULTIPLICATION.

4. Mult. 91738 by 81. 5. 35963 by 72. 6. 847396 by 99.

7. Mult. 38462 by 108. 8. 749357 by 121. 9. 9043278 by 144.

C A S E IV.

When there are cyphers on the right hand of, either the multiplicand, or multiplier, or both, neglect those cyphers; then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

E X A M P L E S.

1. 67910 5600	2. 956700 320	3. 930137000 9500
Prod. 380296000	306144000	8836301500000

4. 359260 7364	5. 8196000 59180	6. 623000 589000
Prod. 2645590640	485039280000	3669470000000

C A S E V.

When there are cyphers between the significant figures of the multiplier, omit multiplying by them, and place the first figure of each product of the significant figures, exactly under that figure by which you multiply; lastly, add them together, and their sum will be the total product.

EXAMPLES.

SIMPLE MULTIPLICATION.

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EXAMPLES.

1.	2.	3.
$\begin{array}{r} 5397 \\ 8009 \\ \hline 48578 \\ 43176 \\ \hline \end{array}$	$\begin{array}{r} 85630 \\ 7005 \\ \hline \end{array}$	$\begin{array}{r} 48976850 \\ 400030 \\ \hline \end{array}$
Prod. 43224578	599838150	19592209305500

CASE VI.

To multiply by 10, 100, 1000, &c. set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.

EXAMPLES.

1.	2.	3.	4.
$\begin{array}{r} 57935 \\ 10 \\ \hline \end{array}$	$\begin{array}{r} 84935 \\ 100 \\ \hline \end{array}$	$\begin{array}{r} 613975 \\ 1000 \\ \hline \end{array}$	$\begin{array}{r} 8473965 \\ 10000 \\ \hline \end{array}$
Prod. 579350			

CASE VII.

To multiply by 99, 999, &c. in one line; place as many dots at the right hand of the multiplicand, as there are figures of 9 in your multiplier, which dots suppose to be cyphers; then, beginning with the right hand dot, subtract the *given* multiplicand from the *new* one, and the remainder will be the total product.

EXAMPLES.

$\begin{array}{r} \text{Mult. } 371967 \dots \\ \text{by } 999 \\ \hline 371595083 \end{array}$	$\left\{ \begin{array}{l} \text{According to the} \\ \text{rule it will} \\ \text{stand thus,} \end{array} \right.$	$\begin{array}{r} 371967 \dots \text{ Minuend.} \\ 371967 \text{ Subtrah.} \\ \hline 371595033 \text{ Rem. or total Prod.} \end{array}$
---	---	---

CASE

22 SIMPLE MULTIPLICATION.

C A S E VIII.

To multiply by 13, 14, 15, &c. to 19 : Place your multiplier at the right of the multiplicand, with the sign of multiplication between them, and multiply with the unit figure, only, of the multiplier, removing the product one figure to the right hand of the multiplicand ; then add all together, and their sum will be the total product.

E X A M P L E S.

$$\begin{array}{r} 75964 \times 13 \\ 227892 \end{array}$$

$$7598 \times 14$$

$$76013 \times 15$$

Prod. 987532

$$227892$$

$$8196 \times 16$$

$$8179 \times 17$$

$$4679 \times 19$$

Prod.

C A S E IX.

To multiply by 101, 102, 103, &c. to 109 : Multiply by the right hand figure, only, of the multiplier, removing the product two figures to the right hand of the multiplicand : Add all together, and the sum will be the total product. In the same manner proceed with 1001, 1002, &c. removing the product three figures.

EXAMPLES.

SIMPLE MULTIPLICATION.

EXAMPLES.

1.

$$\begin{array}{r} 64795 \times 101 \\ 64795 \\ \hline \end{array}$$

Prod. 6544295

2.

$$79164 \times 102$$

3.

$$37598 \times 103$$

4.

$$73967 \times 104$$

5.

$$84973 \times 105$$

6.

$$748794 \times 106$$

Prod.

7.

$$87958 \times 107$$

8.

$$395867 \times 108$$

9.

$$5916379 \times 109$$

Prod.

CASE X.

To multiply by 21, 31, 41, &c. to 91: Multiply by the ten's figure, only, of the multiplier, and set the unit figure of the product under the place of tens; add them all together, and their sum will be the total product.

EXAMPLES.

1.

$$73918 \times 21$$

2.

$$56934 \times 31$$

3.

$$86789 \times 41$$

4.

$$759846 \times 51$$

5.

$$87954 \times 61$$

6.

$$73958 \times 71$$

DIVISION

SIMPLE DIVISION.

D I V I S I O N

Teaches to separate any number, or quantity given, into any number of parts assigned; or to find how often one number is contained in another; or from any two numbers given, to find a third, which shall consist of so many units, as the one of those given numbers is comprehended in the other; and is a concise way of performing several Subtractions.

There are four principal parts to be noticed in Division, viz.

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shews how often the divisor is contained in the dividend.
4. The Remainder (which is always, less than the divisor, and of the same name with the dividend) is very uncertain, as there is sometimes a remainder, and sometimes none.

Division is both simple and compound.

P R O O F.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product; If the work be right, that sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values: As, 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.

CASE.

SIMPLE DIVISION.

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C A S E 1.st

RULE.—First, seek how many times the divisor is contained in a competent number of the first figures of the dividend;—when found, place the figure in the quotient;—multiply the divisor by this quotient figure; place the product under the left hand figures of the dividend; then subtract it therefrom, and bring down the next figure of the dividend to the right hand of the remainder:—If, when you have brought down a figure to the remainder, it is still less than the divisor, a cypher must be placed in the quotient, and another figure be brought down; after which, you must seek, multiply and subtract, till you have brought down every figure of the dividend.

EXAMPLES.

When there is no remainder to a division, the quotient is the absolute and perfect answer to the question; but where there is a remainder, it may be observed, that it goes so much towards another time as it approaches the divisor; thus, if the remainder be half the divisor, it will go half of a time more, and so on; in order therefore to complete the quotient, put the last remainder to the end of it, above a line, and the divisor below it.

It is sometimes difficult to find how often the divisor may be had in the numbers of the several steps of the operation: The best way will be to find how often the first figure of the divisor may be had in the first, or two first figures of the dividend, and the answer, made less by one or two, is generally the figure wanted; but if, after subtracting the product of the divisor and quotient from the dividend, the remainder be equal to, or exceed the divisor, the quotient figure must be increased accordingly.

EXAMPLE 2.

Divisor. Dividend. Quotient.
 $3 \overline{) 175817} \begin{array}{r} 58605 \\ 15 \\ \hline 24 \\ 18 \\ \hline 17 \\ 15 \\ \hline 2 \end{array}$
 Proof. $58605 \times 3 = 175815$
 Quotient. 58605
 Divisor 3
 2 Rem.

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend, therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which, I bring down the next figure of the dividend, viz. 5; then, I inquire how often the divisor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the first.

Observe, that in multiplying by 3, I add in the 2.

EXAMPLES.

SIMPLE DIVISION.

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EXAMPLES.

$$\begin{array}{r} 2. \\ 29 \overline{) 15358(5295} \\ \underline{145} \end{array}$$

$$\begin{array}{r} 85 \\ \underline{58} \end{array}$$

$$\begin{array}{r} 279 \\ \underline{201} \end{array}$$

$$\begin{array}{r} 188 \\ \underline{174} \end{array}$$

$$\begin{array}{r} 14 \end{array}$$

$$\begin{array}{r} 3. \\ 6493 \overline{) 91876375(14150} \\ \underline{6493} \end{array}$$

$$\begin{array}{r} 26946 \\ \underline{25972} \end{array}$$

$$\begin{array}{r} 9743 \\ \underline{6493} \end{array}$$

$$\begin{array}{r} 32507 \\ \underline{32465} \end{array}$$

$$\begin{array}{r} 425 \end{array}$$

$$\begin{array}{r} 4. \\ 28 \overline{) 508775(} \end{array}$$

$$\begin{array}{r} 5. \\ 85 \overline{) 197184(} \end{array}$$

$$\begin{array}{r} 6. \\ 85 \overline{) 994466(} \end{array}$$

$$\begin{array}{r} 7. \\ 236 \overline{) 379867(} \end{array}$$

$$\begin{array}{r} 8. \\ 8479 \overline{) 483956795(} \end{array}$$

$$\begin{array}{r} 9. \\ 5679 \overline{) 19647394(} \end{array}$$

$$\begin{array}{r} 10. \\ 38478 \overline{) 119184693(} \end{array}$$

$$\begin{array}{r} 11. \\ 641976 \overline{) 9187642959(} \end{array}$$

$$\begin{array}{r} 12. \\ 5823789 \overline{) 791822376496(} \end{array}$$

$$\begin{array}{r} 13. \\ 123456789 \overline{) 121932631112635169(} \end{array}$$

CASE II.

When there is one cypher, or more, at the right hand of the divisor, it or they must be cut off; also, cut off the same number of figures from the dividend, and then proceed as in case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.

EXAMPLES.

SIMPLE DIVISION.

EXAMPLES.

$$\begin{array}{r}
 \text{1.} \quad 65 \overline{) 3794326} \quad 2. \quad 5193 \overline{) 8937648} \\
 \underline{325} \\
 544 \\
 \underline{520} \\
 243 \\
 \underline{195} \\
 482 \\
 \underline{455} \\
 276 \\
 \underline{260} \\
 1675 \text{ Rem.}
 \end{array}$$

$$\begin{array}{r}
 \text{3.} \quad 9170 \overline{) 476583} \\
 \text{4.} \quad 878000 \overline{) 91764789430000} \\
 \text{5.} \quad \text{Quot. Rem.} \quad 10 \overline{) 9584} \quad 6. \quad \text{Quot. Rem.} \quad 100 \overline{) 7649580} \quad 7. \quad \text{Quot. Rem.} \quad 1000 \overline{) 98751839462}
 \end{array}$$

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE III.

Short Division is, when the divisor does not exceed 12.

RULE.

First, seek how often the divisor can be had in the first figure, or figures, of the dividend; which, when

SIMPLE DIVISION.

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when found, place in the quotient; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand figures of your dividend, and the units which remain, must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it; then, seek how often you can have your divisor in those two figures; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.
2)71935	3)51923	5)631795	6)8471937
<u> </u>	<u> </u>	<u> </u>	<u> </u>
Quot. 35967—1			
<u> </u>	<u> </u>	<u> </u>	<u> </u>
5.	6.	7.	
8)5137846	9)45968784	11)91843755	
<u> </u>	<u> </u>	<u> </u>	

CASE IV.

When the divisor is such a number, that any two, or more, figures in the Table, being multiplied together, will produce it, divide the given dividend by one of those figures;—the quotient, thence arising, by the other, and so on; and the last quotient will be the answer.*

EXAMPLES.

* As the learner, at present, is supposed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder, I shall here give a rule for finding the true remainder, without having recourse to fractions.

RULE.

SIMPLE DIVISION.

EXAMPLES.

1st Method.

9)196473

8)21830

2d Method.

8)196473

9)24559

Quot. 2728—57

Quot. 2728—57

In the *first* operation, in dividing by 9, 3 remains, and, by 8, 6 remains; which, being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder. In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains, I therefore multiply 7, the last remainder, by 8, adding in the 1, and they make 57, as before.

$$\begin{array}{r}
 36 \overline{) 79638} \\
 25 \overline{) 197835} \\
 84 \overline{) 98975} \\
 54 \overline{) 93738764} \\
 121 \overline{) 75323939} \\
 132 \overline{) 38473692} \\
 144 \overline{) 891376429732}
 \end{array}$$

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is any even part of 100, 1000, &c. is by division; for if the multiplicand be increased by a number of cyphers equal to the places in the multiplier,

RULE.

Multiply the quotient by the divisor; subtract the product from the dividend, and the result will be the true remainder.

The Rule which is most commonly made use of, when the divisor is a composite number, is

RULE II.

Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till you have gone through all the divisors and remainders, to the first.

tiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the product.

1. Mult. 39756 into 125. 2. Mult. 57638 by $33\frac{1}{3}$
 $125 = \frac{1}{8}$ of 1000, wherefore, $33\frac{1}{3} = \frac{1}{3}$ of 100, therefore,
 $8)39756000$ $3)5763800$

4969500 Product.

1921266 $\frac{2}{3}$ Product.

TABLES in COMPOUND ADDITION.

1. M O N E Y.

Note, 4 Farthings } make 1 { Penny. *Marked*
 12 Pence } grs. d.
 20 Shillings } s.
 £.

Farthings.

4 = 1 Penny.
 48 = 12 = 1 Shilling.
 960 = 240 = 20 = 1 Pound.

PENCE TABLES.

d.	s.	d.	d.	s.	d.
20 =	1	8	120 =	10	0
30 =	2	6	130 =	10	10
40 =	3	4	140 =	11	8
50 =	4	2	150 =	12	6
60 =	5	0	160 =	13	4
70 =	6	10	170 =	14	2
80 =	6	8	180 =	15	0
90 =	7	6	190 =	15	10
100 =	8	4	200 =	16	8
110 =	9	2			

s.	d.	s.	d.
1	= 12	11	= 132
2	= 24	12	= 144
3	= 36	13	= 156
4	= 48	14	= 168
5	= 60	15	= 180
6	= 72	16	= 192
7	= 84	17	= 204
8	= 96	18	= 216
9	= 108	19	= 228
10	= 120	20	= 240

TABLES IN COMPOUND ADDITION

2. TROY WEIGHT.*

24 Grains	} make 1	Pennywt.	grs.	pwt.	Marked
20 Pennyweights		Ounce.	oz.		
12 Ounces		Pound.	lb or lb.		

3. AVOIRDUPOIS WEIGHT.†

16 Drains	} make	1 Ounce.	dr. oz.	Marked
16 Ounces		1 Pound.	lb	
28 Pounds		$\frac{1}{4}$ of a hundred wt.	qr.	
4 Quarters		100 wt. or 112 lbs.	Cwt.	
20 Hund. wt.		1 Ton.	T.	

4 APOTHECARIES'

* By this weight are weighed Gold, Silver, Jewels, Eleduaries, and all Liquors.

Note. 175 Troy Ounces are precisely equal to 192 Avordupois Ounces, and 175 Troy pounds are equal to 144 Avordupois. 1 lb Troy = 5760 grains, and 1 lb Avordupois = 7000 grains.

† By Avordupois are weighed all coarse and drilly goods, grocery and chandlery wares; bread, and all metals, except Gold and Silver.

T A B L E S.

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4. APOTHECARIES' WEIGHT.*

60 Grains	}	make 1	{	Scruple. <i>Marked</i> gr.	6
3 Scruples				Dram.	3
8 Drams				Ounce.	3
12 Ounces				Pound.	16

5. CLOTH MEASURE.

2 1/2 Inches	}	make 1	{	Nail. <i>Marked</i> in. na.	
4 Nails, or 9 Inches				Quarter of a yard. qn.	
4 quarters, or 36 in.				Yard.	yd.
3 quarters, or 27 in.				Ell Flemish.	E. Fl.
5 quarters, or 45 in.				Ell English.	E. E.
6 quarters, or 45 in.				Ell French.	E. Fr.
4 qtrs. 1 1/2 in. or 37 in. & 1 fifth				Ell Scotch.	E. Sc.
3 Qrs. & 2 thirds				Spanish Var.	

6. LONG MEASURE.†

3 Barley corns	}	make 1	{	Inch. <i>Marked</i> bar. in.	
12 Inches				Foot.	feet.
3 Feet				Yard.	yd.
5 1/2 Yds. or 16 1/2 feet.				Rod, Perch, or Pole. pol.	
40 Poles				Furlong.	fur.
8 Furlongs				Mile.	mile.
69 1/2 Stat. miles nearly,				Degree of a great Circle	deg.
360 Degrees				A great Circle of the Earth.	

* All the weights now used by Apothecaries, above grains, are Avoirdupois.

The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided.

† The use of long measure is to measure the distance of places, or any other thing, where length is considered without regard to breadth.

Nat.

Or in Measuring Distances.

7200 Inches	} make	1 Link.
25 Links		1 Pole.
100 Links		1 Chain.
10 Chains		1 Furlong.
8 Furlongs		1 Mile.

7. TIME.*

60 Seconds	} make	1 Minute.
60 Minutes		1 Hour.
24 Hours		1 Day.
7 Days		1 Week.
4 Weeks		1 Month.
13 Mo. 1 day & 6 hours		1 Julian year.

8. LAND, or SQUARE MEASURE.

144 Inches	} make	1 Square foot.
9 Feet		— Yard.
30 1/2 Yards, or		— Pole.
272 1/2 Feet		— Rood.
40 Poles		— Acre.
4 Roods, or 160 Roods,		— Mile.
or 4840 yards		
640 Acres		

Note: 60 Geometrical miles make a Degree.—4 Fingers a Hand.—5 Feet a Geometrical Pace. 6 Points make 1 Line, 12 Lines an Inch, 12 Inches a Foot, and 6 Feet one French Toise, or Fathom, equal to 6 Feet 4 Inches, 8,812,875 Lines, English measure. 1 English foot equal to 12 Inches, 2,154 Lines, French.—56 Feet, or 4 Poles, make a Gunter's Chain.—3 Miles make a League.

* By the Calendar, the year is divided in the following manner: Thirty days hath September, April, June and November;

February twenty eight alone, and all the rest have thirty one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap Year, in which February has 29 days.

9. SOLID MEASURE.*

1728 Inches	}	make 1	Foot.
27 Feet			Yard.
40 Ft. of round Tim. or			Ton or Load.
50 ft. of hewn Tim.			
128 Sol. Ft. i. e. 8 in len.			Cord of Wood.
4 in brdth. & 4 in ht.			

10. WINE MEASURE.†

2 Pints.	}	make 1	Quart. Marked	pts. qts.
4 Quarts			Gallon.	gal.
10 Gallons			Anchor of Brandy.	anc.
18 Gallons			Runlet.	run.
31½ Gallons			Half hoghead.	½ hhd.
42 Gallons			Tierce.	tier.
63 Gallons			Hoghead.	hhd.
2 Hogheads			Pipe or Butt.	P. or B.
2 Pipes			Tun.	Tun.

11. ALE OR BEER MEASURE.†

2 Pints	}	make 1	Quart. Marked	pts. qts.
4 Quarts			Gallon.	gal.
8 Gallons			Firk. of Ale in Lon. A. fir.	
8½ Gallons			Firkin of Ale or Beer.	
9 Gallons			Fir. of Beer in Lon. B. fir.	
2 Firkins			Kilderkin.	Kil.
2 Kilderkins			Barrel.	Bar.
1½ Bar. or 45 Gall.			Hoghead of Beer.	hhd.
2 Barrels			Punchon.	pun.
3 Bar. or 4 hhd.			Butt.	Butt.

12. DRY

* By Solid measure are measured all things that have length, breadth and depth.

† All Brandies, Spirits, Perry, Cyder, Mead, Vinegar, Honey, and Oil, are measured by Wine measure; Honey is commonly sold by the pound avoirdupois.

‡ Milk is sold by the Beer quart.

COMPOUND ADDITION.

12. DRY MEASURE.*

2 Pints	} make 1	Quart. <i>Marked</i>	pts. qts
2 Quarts		Pottle.	pot.
2 Pottles		Gallon.	gal.
2 Gallons		Peck.	pk.
4 Pecks		Bushel.	bu.
2 Bushels		Strike.	str.
2 Strikes		Coom.	co.
3 Cooms		Quarter.	qr.
4 Quarters		Chaldron.	ch.
4 Quarters		Chaldron in London.	
5 Quarters		Wey.	wey.
2 Weys		Last.	Last.

COMPOUND ADDITION

Is the adding of several numbers together, having different denominations, as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

R U L E.

1. Place the numbers so that those of the same denomination may stand directly under each other.
2. Add the first column or denomination together as in whole numbers; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column, added, and carry the quotient to the next superior denomination, continuing the same to the last, which add as in simple addition.

1. MONEY.

* This measure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oysters and Coals.

A Winchester Bushel is 28 $\frac{1}{2}$ Inches diameter, and 8 Inches deep.

COMPOUND ADDITION.

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1. MONEY.

EXAMPLES.

1.				2.				3.			
£.	s.	d.		£.	s.	d.	qr.	£.	s.	d.	qr.
9	16	10		47	17	6	2	8	17	11	3
7	10	9		3	9	10	3	4	19	19	1
0	18	6		75	13	9	1	59	6	10	0
5	11	11		4	11	11	0	7	17	16	2
4.				5.				6.			
£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
9	15	10	2	7	6	13	2	5	8	19	3
6	4	8	1	4	3	9	1	7	6	14	1
5	16	11	3	5	9	17	2	9	1	17	2
4	19	2	2	8	17	16	3	1	8	19	3

FRENCH MONEY.

12 Deniers, or pence	H E L P	{	Sol, or Shilling.
20 Sols, or Shillings			Livre, or Pound.
3 Livres, or pounds			Crown of exchange.
6 Livres, or pounds			Real Cro. or ecu d'arg.

Cr. of exc.	liv.	sols.	den.	Ecu d'ar.	liv.	sols.	den.
976	2	17	10	567	5	13	11
379	1	12	6	389	1	19	6
491	1	0	11	548	4	17	10
592	2	15	8	632	3	11	9

DUTCH MONEY.

8 Phennings	H E L P	{	Groat.
2 Groats, or 16 Phennings			Stiver.
20 Stivers			Guilder or Flor.

D

Also,

COMPOUND ADDITION.

A L S O,

12 Groats, or 6 Stivers } make 1 { Schilling.
20 Schillings, or 6 Guilders } Pound.

Guild.	fliv.	gr.	ph.	Guild.	fliv.	ph.	£.	sch.	gro.
167	17	1	7	549	19	14	357	18	11
348	13	0	6	317	16	12	508	12	6
491	13	1	3	859	18	8	497	13	10
749	19	1	4	467	10	15	618	17	8

2. TROY WEIGHT.

1.	2.	3.
℔ oz. pwt. gr.	℔ oz. pwt. gr.	℔ oz. pwt. gr.
767 10 17	649 11 19 20	859 9 15 20
39 6 9	32 9 6 5	487 10 17 22
417 11 16	841 10 11 19	641 11 6 0
935 9 17	478 9 17 23	788 9 12 18

3. AVOIRDUPOIS WEIGHT.

1.	2.	3.
℔ oz. dr.	T. Cwt. grs. ℔	T. Cwt. grs. ℔ oz. dr.
19 13 12	59 13 2 17	91 17 2 25 18 15
21 9 6	6 47 1 21	19 9 0 17 10 12
4 15 15	45 11 3 25	14 13 2 0 9 12
22 10 5	57 16 2 19	47 11 3 19 14 0

4. APOTHECARIES' WEIGHT.

1.	2.	3.
℔ 3 3 gr.	℔ 3 3 gr.	℔ 3 3 3 gr.
9 17	10 7 3 19	12 11 6 1 15
3 2 19	6 3 0 12	4 9 1 0 12
6 1 17	7 6 1 17	91 10 2 2 16
4 0 6	9 5 2 12	4 8 1 2 19

5. CLOTH

COMPOUND ADDITION.

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5. CLOTH MEASURE.

1.	2.	3.
Yd. qr. n.	E. E. qr. n.	E. Fl. qr. n.
70 2 3	91 3 2	914 2 3
3 3 1	49 4 3	49 2 1
42 3 3	6 2 8	561 3 0
57 2 2	84 4 1	84 0 2

6. LONG MEASURE.

1.	2.	3.
Fol. ft. in.	Mil. fur. pol.	Deg. mi. fur. pol. ft. in. br.
12 11 10	9 7 36	759 56 6 29 15 10 2
9 16 9	7 3 19	317 39 1 36 11 6 1
8 12 11	4 1 24	497 63 7 24 9 8 1
7 15 6	6 5 12	562 17 0 11 13 11 0

7. TIME.

1.	2.	3.
W. d. h. m. s.	Y. mo. d.	Y. mo. w. d. h. m. s.
3 6 23 57 42	19 10 19	57 11 3 6 23 29 55
1 5 19 31 28	7 9 27	4 8 1 1 19 45 38
2 3 17 9 15	4 8 16	29 9 2 3 17 18 19
3 0 9 17 58	1 11 14	46 10 2 5 11 50 13

9. LAND or SQUARE MEASURE.

1.	2.	3.
Pol. feet. in.	Yds. ft. in.	Acres. rood. pol. feet. in.
36 179 137	28 7 119	756 3 37 245 128
19 248 119	9 3 75	29 1 28 93 26
12 96 75	29 6 120	416 2 31 128 119
18 110 122	4 8 12	37 1 19 218 20

9. SOLID

COMPOUND ADDITION.

9. SOLID MEASURE.

1.			2.			3.		
Ton.	feet.	in.	Yds.	feet.	in.	Cord.	feet.	in.
29	36	1229	75	22	1412	37	119	1015
12	19	64	9	26	195	9	110	159
8	11	917	3	19	1091	48	127	1073
19	8	1001	28	15	1140	8	101	956

10. WINE MEASURE.

1.				2.				3.			
Tier.	gal.	qts.	pts.	Hhd.	gal.	qts.	pts.	Tun.	hhd.	gal.	qts.
37	39	3	1	51	53	1	1	37	2	37	2
9	17	2	1	27	39	3	0	19	1	59	1
4	28	0	0	9	18	0	1	28	2	0	0
32	19	1	1	0	9	2	1	19	0	47	1

11. ALE and BEER MEASURE.

1.				2.				3.			
A. B.	fir.	gal.		B. B.	fir.	gal.		Hhd.	gal.	qts.	
49	3	7		29	1	8		379	58	3	
26	2	3		19	3	5		19	0	1	
9	0	4		16	0	3		121	37	2	
17	3	0		9	1	8		467	19	1	

12. DRY MEASURE.

1.				2.				3.			
Qrs.	bu.	p.	qt.	Bus.	p.	qt.	pt.	Ch.	bu.	p.	qts.
64	7	3	7	37	2	5	11	37	27	3	5
9	4	1	5	19	8	7	80	6	29	1	7
19	6	2	1	16	2	0	1	15	30	0	0
4	0	2	0	5	1	6	8	4	11	3	0

COMPOUND

COMPOUND SUBTRACTION.

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COMPOUND SUBTRACTION

Teaches to find the difference, inequality, or excess, between any two sums of divers denominations.

R U L E.

Place those numbers under each other, which are of the same denomination, the less being below the greater; begin with the least denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; subtract it therefrom; and to the difference add the upper figure, remembering, always, to add one to the next superior denomination, for that which you borrowed.

1. M O N E Y.

1.				2.			
	£.	s.	d. gr.		£.	s.	d. gr.
Borrowed	849	15	6 1	Lent	791	9	8 1
Paid	195	11	8 1	Received	197	16	4 2
Remains } to pay }	154	3	10 0	Due } to me }			

Proof

3.				4.				5.			
	£.	s.	d. q.		£.	s.	d. q.		£.	s.	d. q.
From	489	9	10 1		843	12	1 3		569	7	5 2
Take	196	0	10 3		746	15	0 2		508	16	4 0

Rem.

d

Borrowed:

COMPOUND SUBTRACTION.

6.				7.			
	£.	s.	d.		£.	s.	d. q.
Borrowed	19372	12	6	Lent	27109	5	8 3
Paid at	223	16	8 0	Rec.	5196	15	10 0
	74	9	7 2		384	17	6 2
fundry	9413	11	0 1	at	4187	18	11 1
	1994	0	10 3	several	1649	16	8 0
	3914	19	0 0		917	9	10 3
times	1064	17	9 1	times	3196	0	2 1
Paid in all				Rec. in all			
Remains to be paid				Rem. due			

2. TROY WEIGHT.

	1.	2.	3.
	lb. oz. pwt. gr.	lb. oz. pwt. gr.	lb. oz. pwt. gr.
Bought	749 5 13 16	374 8 12 10	543 3 9 13
Sold	96 9 19 13	148 4 16 19	179 1 15 18
Rem.			

3. AVOIRDUPOIS WEIGHT.

	1.	2.	3.
	lb. oz. dr.	c. gr. lb.	T. cwt. gr. lb. oz. dr.
Bought	7 9 12	8 2 13	9 11 3 17 5 12
Sold	3 12 9	4 1 15	3 12 1 19 10 9
Rem.			

4. APOTHECARIES'

COMPOUND SUBTRACTION.

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4. APOTHECARIES' WEIGHT.

1.	2.	3.
℥ 3 3 9 gr.	℥ 3 3 9 gr.	℥ 3 3 9 gr.
71 9 3 1 13	65 10 6 2 1	84 1 1 1 1
37 8 4 1 16	31 8 4 2 0	65 9 3 1 17

5. CLOTH MEASURE.

1.	2.	3.	4.
Yds. qr. n.	E. E. qr. n.	E. Fl. qr. n.	E. Fr. qr. n.
35 1 2	467 3 1	765 1 3	549 4 2
19 1 3	291 3 2	149 2 1	197 4 3

6. LONG MEASURE.

1.	2.	3.
Yds. ft. in.	Mil. fur. pol.	Deg. m. fur. p. yds. ft. in. bay.
28 2 10	76 3 11	38 41 3 29 2 1 7 2
17 2 11	27 3 21	19 35 5 31 3 1 9 1

7. TIME.

1.	2.	3.
Mo. d. h. m. s.	Y. mo. d.	Y. mo. w. d. h. m. s.
6 17 13 27 19	7 3 13	48 9 2 5 19 27 31
1 21 16 41 35	4 2 19	19 9 3 4 20 19 49

8. LAND, or SQUARE MEASURE.

1.	2.	3.
A. R. Pol.	A. R. Pol.	A. R. Pol. ft. in.
29 1 10	29 2 17	56 3 19 27 110
24 1 25	17 1 36	29 0 21 210 179

9. SOLID

PROBLEMS.

9. SOLID MEASURE.

1.			2.			3.		
Tons.	ft.	in.	Yds.	ft.	in.	Cords.	ft.	in.
49	19	1100	79	11	917	349	97	1250
38	36	1296	17	25	1095	192	127	1349

10. WINE MEASURE.

1.				2.		3.	
Tun.	gal.	qt.	pt.	Tier.	gal. qt.	Tun.	hhd. gall.
79	1	2	1	19	17 1	532	1 19
38	1	3	1	12	29 2	197	1 47

11. ALE and BEER MEASURE.

1.				2.				3.	
Al. B.	fir.	gal.	qt.	B. B.	fir.	gal.	qts. pts.	Hhds.	gal. qts.
39	1	2	1	21	3	5	2 0	769	17 1
24	3	6	2	19	1	7	2 1	391	42 3

12. DRY MEASURE.

1.				2.				3.			
Qu.	bu.	pk.	qt.	Bu.	pk.	qts.	pts.	Chal.	bu.	pk.	qts.
56	2	2	1	91	1	3	2	39	12	2	1
39	3	1	2	29	2	1	1	24	25	3	2

PROBLEMS.

Resulting from a Comparison of the preceding Rules.

PROB. 1. Having the sum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

Let

Let 288 be the sum of two numbers; one of which is 115, the other is required? From 288 the Sum,
Take 115 the given number,
Rem. 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other, 198: What is the other? From 325 the greater,
Take 198 the difference,
Rem. 127 the less.

PROB. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the less number.
198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule. To half the sum add half the difference, and the sum is the greater, and from half the sum take half the difference, and the remainder is the less.—Or, from the sum take the difference, and, half the remainder is the least: To the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

$$2)48$$

$$2)14$$

$$\frac{1}{2} \text{ Sum} = 24 \quad \frac{1}{2} \text{ diff.} = 7$$

$$24 + 7 = 31 \text{ the greater, and } 24 - 7 = 17 \text{ the less.}$$

$$\text{Or } 48 - 14 \div 2 = 17, \text{ and } 17 + 14 = 31.$$

PROB.

PROB. 5. Having the sum of two numbers and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: You will then have their sum and difference to find the numbers by Prob. 4.

What two numbers are those, whose sum is 32, and the difference of whose squares is 256?

	Half sum	16
	Half diff,	4
		—
32)256(8 difference.	Greater	20
256		—
—	Less	12

PROB. 6. Having the difference of two numbers and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum, then proceed by Prob. 4.

What are those two numbers, whose difference is 20, and the difference of whose squares is 2000?
 20)2000(100 Sum. $50+10=60$, the greater, and
 $50-10=40$, the less.

PROB. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let

- P R O B L E M S. 47

Let the product of two numbers be 288, $\left\{ \begin{array}{l} 8)288 \\ \hline \end{array} \right.$ and one of them 8; I demand the other? $\left. \begin{array}{l} \\ \end{array} \right\} \text{Ans. } 36$

PROB. 8. Having the dividend and quotient, to find the divisor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Division*.

Given $\left\{ \begin{array}{l} 288 \text{ the Dividend.} \\ 36 \text{ the Quotient.} \end{array} \right.$ $\begin{array}{r} 36)288(8 \text{ Divisor.} \\ \underline{288} \end{array}$
Required the Divisor?

PROB. 9. Having the Divisor and Quotient given, to find the Dividend.

Rule. Multiply them together.

Given $\left\{ \begin{array}{l} 8 \text{ the Divisor.} \\ 36 \text{ the Quotient.} \end{array} \right.$ $\begin{array}{r} 85 \\ 8 \\ \hline \end{array}$
Required the Dividend? $\underline{288 \text{ the Dividend.}}$

By a due consideration and application of these Problems only, many questions may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

APPLICATION of the preceding Rules.

1. The least of two numbers is 19418, and the difference between them is 238; What is the greater, and sum of both?

$19418 + 238 = 21802$ greater, and $19418 + 21802 = 41220$ sum.

2. Suppose a man born in the year 1743; When will he be 57 years of age?

$1743 + 57 = 1800$ Ans.

What

3. What number is that, which being added to 19418, will make 21802 ?

$21802 - 19418 = 2384$, *Ans.*

4. General *Washington* was born in 1732 ; What is his age in 1787 ?

$1787 - 1732 = 55$ *Ans.*

5. America was discovered by *Columbus* in 1492, and its Independence declared in 1776 ; How many years have elapsed between those two Eras ?

$1776 - 1492 = 284$ *Ans.*

6. The Massacre at Boston, by the British Troops, happened, March 5th, 1770, and the Battle of Lexington, April 19th, 1775 : How long between ?

April 19th, 1775—March 5, 1770 = 5y. 1m. 14d. *Ans.*

7. General *Burgoyne* and his Army were captured October 17th, 1777, and Earl *Cornwallis* and his Army, October 19th, 1781 : What space of time between ?

October 19th, 1781—October 17th, 1777 = 4 years and 2 days, *Ans.*

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783 ; How long did the war continue ?

January 20th, 1783—April 19th, 1775 = 7y. 9m. 1d. *Ans.*

9. A, B, C, and D, purchased a quantity of goods in partnership ; A paid £12 10s. a dollar and a crown piece ; B 35s. C 29s. 10d. and D 79d. What did the goods cost ?

$\text{£}16$ 14s. 1d. *Ans.*

10. A man borrowed, at different times, these several sums, viz, £29 5s. £18 17s. 6d. £45 12s. £98 3 dollars, one crown piece and an half ; Pray how much was he in debt ?

Ans. £198 2s. 6d.

11. There

11. There are 4 numbers ; the first 317, the second 912, the third 1829, and the fourth as much as the other three, abating 97 : What is the sum of them all ?

Ans. 4819.

12. Bought a quantity of Goods for £125 10s. paid for truckage 45s. for freight 79s. 6d. for duties 35s. 10d. and my expenses were 53s. 9d : What did the Goods stand me in ?

Ans. £136 4s. 1d.

13. A Gentleman left his son £1725 more than his daughter, whose fortune was 15 thousand 15 hundred and 15 pounds : What was the son's portion, and what did the whole estate amount to ?

Ans. The son's fortune, £18240, and the whole estate £34755.

14. A merchant had 6 debtors, who, together, owed him £4917 10s. 6d. A, B, C, D and E, owed him £1675 13s. 9d. of it : What was F's debt ?

Ans. £1241 16s. 9d.

15. What is the difference between £1309 7s. 1d. and the amount of £345 13s. 4d. and £571 4s. 8d. ?

Ans. £392 9s. 1d.

16. A Merchant, at his first engaging in trade, owed £937 15s. he had in cash £1755 3s. 6d. in goods £459 12s. 3d. in good debts £197 16s. and he cleared the first year £249 19s. 10d. What was the neat balance at the year's end ?

Ans. £1724 16s. 7d.

17. What sum of money must be divided between 12 men, so as that each may receive £155 ?

$£155 \times 12 = £1860$ *Ans.*

18. What number must I multiply by 9, that the product may be 675 ?

$675 \div 9 = 75$ *Ans.*

19. A Privateer of 175 men took a prize, which amounted to £59 per man, beside the owner's half : What was the value of the prize ?

$175 \times 59 \times 2 = £20650$ *Ans.*

E

20. What

20. What is the difference between thrice five, and thirty ; and thrice thirty five ?

$$35 \times 3 - 5 \times 3 + 30 = 60, \text{ Ans.}$$

21. The sum of two numbers is 750 ; the less 248 : What is the difference, product, and the square of their difference ?

$$750 - 248 = 502 \text{ the greater number, } 502 - 248 = 254 \text{ difference, } 502 \times 248 = 124496 \text{ product, and } 254 \times 254 = 64516 \text{ square of the difference.}$$

22. What is the difference between six dozen dozen, and half a dozen dozen ; and what is their product, and the quotient of the greater by the less ?

$$\text{Ans. } 6 \times 12 \times 12 - 6 \times 12 = 792 \text{ diff. } 6 \times 12 \times 12 \times 6 \times 12 = 62208 \text{ product, and } 6 \times 12 \times 12 \div 6 \times 12 = 12, \text{ Quotient.}$$

23. There are two numbers ; the greater of them is 25 times 78, and their difference is 9 times 25 ; their sum and product are required.

$$\text{Ans. } 78 \times 25 = 1950 \text{ the greater, } 1950 - 15 \times 9 = 1815 \text{ the less. } 1950 + 1815 = 3765 \text{ the sum, and } 1950 \times 1815 = 3539250 \text{ the product.}$$

24. A Merchant began trade with £25327 ; for 6 years together, he cleared £1253 *per annum* ; the next 5 years, he cleared £1729 *per annum* ; but, the last 4 years, had the misfortune to lose £3019 *per annum* : What was he worth at the 15 years' end ?

$$\text{Ans. } £29414.$$

25. If a man spends £192 in a year : What is that per Calendar month ?

$$192 \div 12 = £16 \text{ Ans.}$$

26. If

* A Number is said to be squared, when it is multiplied to itself.

REDUCTION.

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26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States: What will be the share of each? *Ans.* $3230769\frac{2}{3}$ dollars.

27. If 5000 men march in a column of 750 deep: How many march abreast? *Ans.* $9000 \div 750 = 12$

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the same? *Ans.* $3072 \div 32 - 32 = 64$

29. What number is that, which, multiplied by 3589, will produce 92050672? *Ans.* $92050672 \div 3589 = 25648$

30. Suppose the quotient arising from the division of two numbers to be 5379, the divisor 37625: What is the dividend, if the remainder come out 9357? *Ans.* $37625 \times 5379 + 9357 = 202391232$

REDUCTION

Teaches to bring, or exchange, numbers of one denomination to others of different denominations, retaining the same value.

It is of two sorts, viz. Descending and Ascending; the former of which is performed by Multiplication, and the latter, by Division.

REDUCTION DESCENDING.

RULE.

Multiply the highest denomination, given, by so many of the next less, as make one of that greater, and thus continue till you have brought it down as low as your question requires.

PROOF. Change the order of the question, and divide your last product by the last multiplier, and so on.

EXAMPLES.

REDUCTION.

EXAMPLES.

1. In £27 15s. 9d. 2qrs. how many farthings?

f. s. d. qrs.

27 15 9 2

Multiplied by 20 = Shillings in a pound.

555 = Shillings.

by 12 = Pence in a shilling.

6669 = Pence.

by 4 = Farthings in a penny.

Ans. = 26678 Farthings.

Note. In multiplying by 20, I added in the 15s. — by 12, the 9d. and by 4, the 2qrs. which must always be done in like cases.

To prove the above question, change the order of it, and it will stand thus: In 26678 farthings, how many pounds?

4)26678

12)6669 2 qrs.

2)0)55)5 9d.

Answer, £27 15 9 2.

2. In £36 12s. 10d. 1qr. how many farthings?

Ans. 86177.

3. In £95 11s. 5d. 3qrs. how many farthings?

Ans. 91751.

4. In £719 9s. 11d. how many half pence?

Ans. 345358.

5. In 29 guineas, at 28s. how many pence?

Ans. 9744.

REDUCTION.

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6. In 37 pistoles, at 22s. how many shillings, pence and farthings?

Ans. 814s. 9768d. 39072 farthings.

7. In 49 half johannes, at 48s. how many six pences?

Ans. 4704.

8. In 473 French crowns, at 6s. 8d. how many threepences?

Ans. 12613½.

9. In 58 moidores, at 36s. how many shillings, pence and farthings?

Ans. 1908s. 22896d. 91584grs.

10. In £29 how many groats, threepences, pence and farthings?

Ans. 1740 groats, 2320 threepences, 6960d. 27840grs.

11. Reduce 47 guineas and one fourth of a guinea into shillings, sixpences, groats, threepences, twopences, pence and farthings.

Ans. 1323 shillings, 2646 sixpences, 3969 groats, 5292 threepences, 7938 twopences, 15876 pence, and 63504 farthings.

REDUCTION ASCENDING.

RULE.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your question requires.

EXAMPLES.

1. In 547325 farthings how many pence, shillings and pounds?

Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1q.

Shillings in a pound = 20)11402 7d.

£570 2s. 7d. 1qr.

Ans. 136831d. 11402s. and £570.

Ans.

Note. The remainder is always of the same name as the dividend.

2. Bring 35177 farthings into pounds.
3. Bring 91751 farthings into pence, &c.
4. Bring 345358 halfpence into pence, shillings and pounds.
5. Reduce 9744 pence to guineas, at 28s. per guinea.
6. In 39072 farthings, how many pistoles, at 22s. ?
7. In 4704 sixpences, how many half johannes ?
8. In 126134, threepences, how many French crowns, at 6s. 8d. ?
9. In 91584 farthings, how many moidores, at 36s. ?
10. In 27840 farthings, how many pence, threepences, groats, shillings and pounds ?
11. In 63504 farthings, how many pence, twopences, threepences, groats, sixpences, shillings and guineas ?

Note. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING and ASCENDING.

1. M O N E Y.

1. In £97 how many pence, and English or French crowns, at 6s. 8d. ?

Ans. 23280d. and 291 crowns.

2. In 947 English crowns, at 6s. 8d. how many shillings, and English guineas ?

Ans. 6313s. 4d. and 225 guin. 13s. 4d.

3. In

3. In 519 English half crowns, how many pence and pounds ? *Ans.* 20760^l. and £86 10s.

4. In 1259 groats, how many farthings, pence, shillings and guineas ?

Ans. 20144^{grs}. 5036^d. 419^s. 8^d. and 14 guineas, 27^s. 8^d.

5. In 75 pistoles, how many pounds ?

Ans. £82 10s.

6. In 735 French crowns, how many shillings and French guineas, at 26^s. 8^d. ?

Ans. 4900s. and 183 guineas, 20s.

7. In 5793 pence, how many farthings, pounds and pistoles ?

Ans. 23172^{qrs}. £24 2s. 9^d. and 21 pistoles, 20s. 9^d.

8. In £99 how many shillings, and half johannes, at 48^s. ? *Ans.* 1980s. and 41 half joes, 12s.

9. In £179 how many guineas ?

Ans. 127 guineas, 24s.

10. In £345 how many moidores ?

Ans. 191 moidores 24s.

11. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 English crowns, and 19 dollars ; how many pounds, half joes, moidores, guineas, pistoles, English crowns, dollars, shillings, pence and farthings ?

Ans. £354 4s. 147 half joes, 28s. 196 moidores, 28s. 253 guineas, 322 pistoles, 1062 English crowns, 4s. 1180 dollars, 4s. 7084 shillings, 85008^d. and 340032^{grs}.

When it is required to know how many sorts of coin, of different values, and of equal number, are contained in any number of another kind ; reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor ; then, reduce the money given, into the same denomination

denomination for a dividend, and the quotient arising from the division, will be the number required.

Note. Observe the same direction in weights and measures.

1. In 275 half johannes, how many moidores, guineas, pistoles, dollars, shillings and six pences, of each, the like number?

A moid. is 36s. } 72 sixpences. 275 half joes.
that is 48 shill. in a johan.

A guin. is 28s. } 56 ditto. 2200
that is 1100

A pistole is 22s. } 44 ditto. 13200 shillings.
that is 2 sixp. in a shill.

A dollar is 6s. } 12 do. Div. = 26400 sixpences.
that is

One shilling has 2

1

Divisor = 187 sixpences.

187)26400(141 of each, and 33 sixpences, or 16s. 6d. over, the *Answer*.

2. A Gentleman distributed £37 10s. between 4 persons in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third, 10s. and the fourth, 5s. What did each person receive?

Ans. the first man £15.

REDUCTION.

57

2. TROY WEIGHT.

1. How many grs. in a silver bowl, that weighs 3 lb. 10 oz. 12 pwt.?

lb. oz. pwt.

3 10 12

12 ounces in a pound.

46 ounces.

20 pennyweights in one ounce.

932 pennyweights.

24 grains in one pwt.

3728

1864

Proof. 24)22368 grains, Answer.

2|093|2

12)46—12 pwt.

lb. 3—10 oz.

2. In 487 ozs. how many pwt. and grs.?

Ans. 9740 pwt. 233760 grs.

3. In 13 ingots of gold, each weighing 9 oz. 5 pwt. how many grains?

Ans. 57720 gr.

4. In 97397 grs. how many pounds?

Ans. 16 lb. 10 oz. 18 pwt. 5 gr.

5. How many rings, each weighing 5 pwt. 7 gr. may be made of 3 lb. 5 oz. 16 pwt. 2 gr. of gold?

Ans. 158.

3. AVOIRDUPOIS.

REDUCTION.

3. AVOIRDUPOIS.

Cwt. qr. lb. oz.

1. In 91 3 17 14 how many ounces?

4

367 quarters.

Proof.

28

16)164702

2943

28)10293 14oz.

735

367 17lb.

10293 pounds.

16

Cwt. 91 3qrs.

61762

10294

164702 ounces.

2. In 12 tons, 15cwt. 1qr. 19lb. 6oz. 12dr. how many drams?

Ans. 7323500dr.

3. In 24lb 11oz. 9dr. how many drams?

Ans. 6329dr.

4. In 44800 pounds, how many drams and tons?

Ans. 11468800dr. and 20 tons.

REDUCTION

59

5. In 28^{lb} Avoirdupois, how many pounds Troy?

^{lb}

28

7000 grains, in 1^{lb} Avoirdupois.

grs. in } = 576|0)19600|0(34^{lb}.
1^{lb} tr.

1728

2320

2304

160

12

576|0)192|0(00z.

20

576|0)3840|0(6^{pwt.}

3456

3840

24

1536

768

576|0)9216|0(16^{gr.}

576

3456

3456

6. In 47^{lb} 9oz. 13^{pwt.} 17^{grs.} Troy, how many ^{lb} Avoirdupois?

^{lb} oz. ^{pwt.} ^{gr.}

47 9 13 17

12

572

20

11473

24

45899

27947

7|000)275|369(39^{lb}.

21

65

63

2369

18

14814

2369

7|000)27|904(50z.

(Carried over)

REDUCTION.

Brought over 7|000)37|904(502.

35

2904

16

17424

2904

7|000)46|464(64464 dr.

42

4464

4. APOTHECARIES' WEIGHT.

1. How many grains are there in 37lb 63?

lb oz.

37 6

12

Proof.

2|0)21600|0

3)10800

450 ounces.

8

8)3600

3600 drams.

8

12)450

37lb 63

10800 scruples.

20

216000 grains.

2. In 9lb 83 13 29 19gr. how many grains?

Ans. 55799gr.

3. In 55799 grains, how many pounds, &c.

Ans. 9lb 83 13 29 19gr.

5. CLOTH

REDUCTION.

61

5. CLOTH MEASURE.

1. In 127 yards, how many quarters and nails?

Yds.

127

4

—

508 qrs.

4

—

Ans. 2032 nails.

Proof.

4)2032

4)508

127 yards.

2. In 9173 nails, how many yards?

Ans. 573 yds. 1 qr. 1 n.

3. In 75 ells English, how many quarters and nails?

Ans. 375 qrs. 15 con.

4. In 56 ells Flemish, how many quarters and nails?

Ans. 168 qrs. 672 n.

5. In 39 ells French, how many quarters and nails?

Ans. 234 qrs. 936 n.

6. In 723 nails, how many yards, ells Flemish, ells English and ells French?

Ans. 453 yds. 604 ells Flem. 862 ells Eng. 2 qrs. 302 ells French.

7. In 19 pieces of cloth, each 15 yards 2 quarters, how many yards, quarters and nails?

Ans. 294 yds. 218 qrs. 17 n.

F

6. LONG

REDUCTION.

6. LONG MEASURE.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

Miles.

43
8

Proof.

8)8173440

344 furlongs.

12)2724480

40

3)227040

13760 rods.

11)75680

5½

6880

68800

6880

2

75680 yards.

40)13760

8

8)344

227040 feet.

12

43

2724480 inches.

3

8173440 Answer.

Here I divide by 11, and multiply the quotient by 2, because twice 5½ is 11,—or I might first have multiplied by 2, and, then, have divided the product by 11.

2. How many barley corns will reach round the globe, it being 360 degrees?

Ans. 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles?

Ans. 171071000.

How

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles?

Ans. 13440 times.

5. In 190080 inches, how many yards and leagues?

Ans. 5280 yds. and 1 league.

7. TIME.

1. In 20 years, how many seconds?

d. h.
365 6 in a year.

24

1466

730

8766 hours in 1 year.

20

175320 hours in 20 years.

60

10519200 minutes in ditto.

60

631152000 seconds in ditto.

Proof.

6|0)63115200|0

6|0)1051920|0

2|0)17532|0

4X5)8766

4)1461

365d. 6h.

2. Suppose your age to be 15y. 19d. 11h. 37m. 45s. how many seconds are there in it, allowing 365 days and 6 hours to the year?

Ans. 475047465.

3. How many minutes from the first day of January to the 14th day of August, inclusively?

Ans. 325440.

4. How many days since the commencement of the Christian Era?

5. How

5. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?

6. How many seconds between the commencement of the war, April 19th, 1775, and the Independence of the United States of America, which took place the 4th day of July, 1776?

Ans. 38102400.

8. LAND or SQUARE MEASURE.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

Acr. r. p.
29 3 19
4

119 roods.

40

Ans. 4779 perches.

Proof.

40)4779

4)119—19p.

19ac.—3r.

2. In 1997 poles, how many acres?

Ans. 12a. 1r. 37p.

9. SOLID MEASURE.

1. In 15 tons of hewn timber how many solid inches?

15 tons. *Proof.*

50

750 feet.

1728

6000

1500

5250

750

Ans. 1296000 inches.

1728)1296000(750

12096

15 tons.

8640

8640

2. In

REDUCTION.

65

2. In 9 tons of round timber, how many inches?
Ans. 622080.

10. WINE MEASURE.

1. In 9hhd. 15gal. 3qts. of wine, how many quarts?

hhd. gal. qts.

9 15 3

63

—

32

55

—

582 gallons.

4

—

Proof.

4)2331

—

63)582—3qts.

—

9hhd.—155.

Ans. 2331 quarts.

2. In 12 pipes of wine, how many pints?

Ans. 12096.

3. In 9758 pints of brandy, how many pipes?

Ans. 9p. 1hhd. 22gal. 3qts.

4. In 1008 quarts of cider, how many tons?

Ans. 1 ton.

11. ALE or BEER MEASURE.

1. In 29hhd. of beer, how many pints?

hhd.

29

54

—

116

145

—

1566 gallons.

4

—

6264 quarts.

2

—

Proof.

2)12528

—

4)6264

—

54)1566

—

29hhd.

Ans. 12528 pints.

f

14

2. In 47 *bar*, 18 *gal*, of ale, how many pints?

Ans. 12928.

3. In 36 puncheons of beer, how many butts?

Ans. 24.

12. DRY MEASURE.

1. In 42 chaldrons of coals, how many pecks?

Chaldrons

Proof.

42

4)5376

32

32)1344(42

—

128

84

126

64

—

—64

1344 bushels.

Ans. 5376 pecks.

2. In 75 bushels of corn, how many pints?

Ans. 4800.

3. In 9376 quarts, how many bushels?

Ans. 293.

VULGAR FRACTIONS.

Fractions, or broken numbers, are expressions for any assignable parts of an unit, or whole number; and are represented by two numbers, placed one above another, with a line drawn between them, thus, $\frac{5}{8}$, &c. signifying five eighths, four thirds, that is one and one third, &c.

The figure, above the line, is called the *numerator*, and that below it, the *denominator*.

The denominator (which is the divisor in division) shews how many parts the integer is divided into; and the numerator (which is the remainder

after division) shews how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed. Any whole number may be made an improper fraction, by drawing a line under it, and putting unity, or 1 for a denominator, as 9 may be expressed fractionwise, thus, $\frac{9}{1}$, and thus, $\frac{12}{4}$, &c.

1. A *single, simple, or proper fraction* is, when the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{12}{15}$, &c. and is a *simple* expression for any number of parts of the integer.

2. An *improper fraction* is, when the numerator exceeds the denominator, as $\frac{5}{3}$, $\frac{7}{4}$, $\frac{12}{5}$, &c.

3. A *compound fraction* is the fraction of a fraction, coupled by the word *of*, thus, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$, &c. which are read thus, two thirds of three fourths; one half of three fifths of seven eighths.

4. A *mixed number* is composed of a whole number and a fraction, as $7\frac{3}{4}$, $85\frac{1}{2}$, &c. that is, seven and three fourths, &c.

5. A fraction is said to be in its least, or lowest terms, when it is expressed by the least numbers possible.

6. The *common measure* of two, or more numbers, is that number which will divide each of them, without a remainder: Thus, 5 is the common measure of 10, 20 and 30; and the *greatest* number, which will do this, is called the *greatest common measure*.

7. A number, which can be measured by two, or more numbers, is called their *common multiple*; and, if it be the least number, which can be so measured, it is called the *least common multiple*: thus, 40, 60, 80, 100 are multiples of 4 and 5; but their least common multiple is 20.

8. A *prime number* is that, which can only be measured by itself, or an unit.

9. That number, which is produced by multiplying several numbers together, is called a *composite number*.

10. A *perfect number* is equal to the sum of all its aliquot parts.

PROBLEM I.

To find the greatest common measure of two, or more numbers.

RULE.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before; then, of that common measure and one of the other numbers, and so on, through all the numbers, to the last; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

EXAMPLES.

VULGAR FRACTIONS.

69

EXAMPLES.

1. What is the greatest common measure of 1836, 3996, and 1044?

So 108 is the greatest common measure of 3996 and 1836

$$\begin{array}{r} 1836 \overline{) 3996} (2 \\ \underline{3672} \end{array}$$

Hence 108)1044(9

$$\begin{array}{r} 324 \overline{) 1836} (5 \\ \underline{1620} \end{array}$$

$$\begin{array}{r} 972 \overline{) 1044} (1 \\ \underline{972} \end{array}$$

$$\begin{array}{r} 216 \overline{) 324} (1 \\ \underline{216} \end{array}$$

$$\begin{array}{r} 72 \overline{) 108} (1 \\ \underline{72} \end{array}$$

$$\begin{array}{r} 72 \overline{) 72} (1 \\ \underline{72} \end{array}$$

Com. meas. = 108)216(2
216
—

Last gr. com. meas. = 36)72(2

$$\begin{array}{r} 72 \overline{) 72} (1 \\ \underline{72} \end{array}$$

Therefore, 36 is the answer required.

2. What is the greatest common measure of 1224 and 1080?

Ans. 72.

PROBLEM II.

To find the least common multiple of two, or more numbers.

RULE.

1. Divide by any number that will divide two or more, of the given numbers, without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2. Divide the second line, as before, and so on, till there are no two numbers, that can be divided; then, the continued product of the divisors and quotients, will give the multiple required.

EXAMPLES.

EXAMPLES.

1. What is the least common multiple of 6, 10, 16 and 20?

$$\begin{array}{r} *5)6 \quad 10 \quad 16 \quad 20 \\ \hline \end{array}$$

$$\begin{array}{r} *2)6 \quad 2 \quad 16 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} *2)3 \quad 1 \quad 8 \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} *3 \quad 1 \quad *4 \quad 1 \\ \hline \end{array}$$

I survey my given numbers and find that five will divide two of them, viz. 10 and 20, which I divide by 5, bringing, into a line with the quotients, the numbers, which 5 will not measure : Again, I view the numbers in the second line, and find 2 will measure them all, and I get 3, 1, 8, 2, in the third line, and find that 2 will measure 8 and 2, and in the fourth line, get 3, 1, 4, 1, all prime, I then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. What is the least common multiple of 6 and 8?

Anf. 24.

3. What is the least number that 3, 5, 8 and 10 will measure?

Anf. 120.

4. What is the least number which can be divided by the 9 digits, separately, without a remainder?

Anf. 2520.

REDUCTION of VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE

VULGAR FRACTIONS.

71

C A S E I.

To abbreviate, or reduce fractions to their lowest terms,

R U L E.

Divide the terms of the given fraction by any number, which will divide them without a remainder, and the quotients, again, in the same manner; and so on, till it appears that there is no number greater

* That dividing both the terms, that is, both numerator and denominator of the fraction, equally by any number whatever, will give another fraction, equal to the former, is evident: And if those divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. Any number, ending with an even number or cypher, is divisible by 2.

2. Any number, ending with 5 or 0, is divisible by 5.

3. If the right hand place of any number be 0, the whole is divisible by 10.

4. If the two right hand figures of any number be divisible by 4, the whole is divisible by 4.

5. If the three right hand figures of any number be divisible by 8, the whole is divisible by 8.

6. If the sum of the digits, constituting any number, be divisible by 3 or 9, the whole is divisible by 3 or 9.

7. If a number cannot be divided by some number less than the square root thereof, that number is a *prime*.

8. All *prime* numbers, except 2 and 5, have 1, 3, 7, or 9 in the place of units; and all other numbers are *composite*.

9. When numbers, with the sign of Addition or Subtraction between them, are to be divided by any number, each of the numbers must be divided: Thus, $6+9+12 = 3+3+4 = 9$.

10. But if the numbers have the sign of Multiplication between them; then only one of them must be divided: Thus,

$$4 \times 6 \times 10 = 2 \times 6 \times 10 = 2 \times 6 \times 5 = 24 = 24$$

11. If the numbers have the sign of Division between them, only one of them must be divided: Thus,

greater than 1, which will divide them, and the fraction will be in its lowest terms. *Or,*

Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

E X A M P L E S.

1. Reduce $\frac{288}{192}$ to its lowest terms.

$$8 \left\{ \begin{array}{l} (4) \\ (3) \end{array} \right. \frac{288}{192} = \frac{36}{24} = \frac{3}{2} = \frac{3}{2} \text{ the answer.}$$

Or thus :

$\begin{array}{r} 288 \overline{) 180} \\ 288 \end{array}$ Therefore 96 is the greatest common measure.

$\begin{array}{r} 192 \overline{) 288} \\ 192 \end{array}$ and $96 \left\{ \frac{288}{192} = \frac{3}{2} \text{ the same as before.}$

$\begin{array}{r} 192 \\ \hline \end{array}$ Com. meas. 96 $\begin{array}{r} 192(2 \\ 192 \end{array}$

2. Reduce $\frac{57}{19}$ to its lowest terms.

Ans. $\frac{57}{19}$

3. Reduce $\frac{46}{19}$ to its lowest terms.

Ans. $\frac{46}{19}$

4. Reduce $\frac{142}{19}$ to its lowest terms.

Ans. $\frac{142}{19}$

C A S E II.

To reduce a mixed number to its equivalent improper Fraction.

R E G U L A.

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

E X A M P L E S.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4 is $\frac{3}{4}$.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ 8 \div 5 \\ \hline \text{Ans. } 293 \\ \hline 8 \\ \hline \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum 293 is the numerator of the fraction sought, and 8 the denominator : So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{36 \times 8 + 5}{8} = \frac{293}{8}$ Answer as before.

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction.

Ans. $\frac{2163}{17}$

3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction.

Ans. $\frac{12510}{19}$

CASE III.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator : Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8.

$6 \times 8 = 48$, and $\frac{48}{8}$ the Ans.—Proof $\frac{48}{8} = 48 \div 8 = 6$.

Q

2. Reduce

2. Reduce 15 to a fraction, whose denominator shall be 12 .

Ans. $\frac{180}{12}$.

C A S E IV.

To reduce an improper fraction to its equivalent whole, or mixed number.

R U L E.

Divide the numerator by the denominator; the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

E X A M P L E S.

1. Reduce $\frac{243}{8}$ to its equivalent whole, or mixed number.

8)293($36\frac{5}{8}$ Ans.

$$\begin{array}{r} 24 \\ \hline 53 \\ 48 \\ \hline 5 \end{array}$$

Or, $\frac{243}{8} = 293 \div 8 = 36\frac{5}{8}$ as before.

2. Reduce $\frac{2157}{17}$ to its equivalent whole, or mixed number.

Ans. $127\frac{3}{17}$.

3. Reduce $\frac{81}{9}$ to its equivalent whole number.

Ans. 9.

C A S E V.

To reduce a compound fraction to an equivalent simple one.

R U L E.

Multiply all the numerators continually together for a new numerator, and all the denominators, for

* This rule is evidently the reverse of case 2d.

for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged, and the other members continually multiplied, (as by the rule) will produce the fractions required in lower terms.

E X A M P L E S.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1 \times 2 \times 3 \times 4}{2 \times 3 \times 4 \times 5} = \frac{24}{120} = \frac{1}{5} \text{ the Answer.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ to a simple fraction.

$$\frac{3 \times 4 \times 5 \times 6}{4 \times 5 \times 6 \times 7} = \frac{360}{840} = \frac{3}{7} \text{ Ans. Or, by expunging}$$

the equal numerators and denominators, it will

$$\text{be } \frac{3 \times 6}{6 \times 7} = \frac{32}{71} = \frac{3}{7} \text{ as before.}$$

C A S E VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

R U L E I.

Multiply each numerator into all the denominators, except its own, for a new numerator, and all the

the denominators into each other, continually, for a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{3}{8}$ to equivalent fractions, having a common denominator.

$$1 \times 5 \times 8 = 40 \text{ the new numerator for } \frac{1}{4}.$$

$$2 \times 4 \times 8 = 64 \text{ ditto for } \frac{2}{5}.$$

$$5 \times 4 \times 5 = 100 \text{ ditto for } \frac{3}{8}.$$

$$4 \times 5 \times 8 = 160 \text{ the common denominator.}$$

Therefore the new equivalent fractions are $\frac{10}{160}$, $\frac{64}{160}$ and $\frac{100}{160}$, the *Ans.*

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ to fractions having a common denominator.

$$\text{Ans. } \frac{576}{1152}, \frac{768}{1152}, \frac{864}{1152}, \frac{960}{1152}, \frac{1008}{1152}.$$

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{1}{2}$, and $\frac{1}{3}$, to a common denominator.

$$\text{Ans. } \frac{936}{1872}, \frac{1040}{1872}, \frac{14508}{1872}, \frac{432}{1872}.$$

RULE II.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Prob. 2, page 70, find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

VULGAR FRACTIONS.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{8}$ to fractions, having the least common denominator possible.

$$\begin{array}{r} 4) 3 \quad 4 \quad 8 \\ \hline 3 \quad 1 \quad 2 \\ \hline \end{array}$$

$$4 \times 3 \times 2 = 24 = \text{least common denominator.}$$

$24 \div 3 \times 1 = 8$ the first numerator; $24 \div 4 \times 3 = 18$ the second numerator; $24 \div 8 \times 3 = 3$ the third numerator.

Whence, the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{3}{24}$.

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

RULES.

Reduce the given fraction to a compound one by comparing it with all the denominations between it and that denomination you would reduce it to; lastly, reduce this compound fraction to a single one, by case 5th, and you will have a fraction of the required denomination, equal in value to the given fraction.

EXAMPLES.

1. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound.

By comparing it, it becomes $\frac{3}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$; which, reduced by case 5th, will be $3 \times 1 \times 1 = 3$.

$$\text{And } 5 \times 12 \times 20 = 1200 = \frac{1}{1200}$$

g

2. Reduce

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound. *Ans.* $\frac{1}{1120}$
3. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea. *Ans.* $\frac{1}{224}$ guinea.
4. Reduce $\frac{1}{2}$ of a shilling to the fraction of a moidore. *Ans.* $\frac{1}{5}$ moidore.
5. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb Avoirdupois. *Ans.* $\frac{1}{14}$ lb.
6. Reduce 3s. 6d. to the fraction of a pound. *Ans.* $\frac{7}{20}$
7. Reduce 13s. 6d. to the fraction of a pistole. *Ans.* $\frac{27}{44}$ pistole.
8. *Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea. *Ans.* $\frac{4}{7}$ guinea.
9. Reduce $\frac{7}{8}$ of a pwt. to the fraction of a pound Troy. *Ans.* $\frac{7}{1920}$ lb.
10. Reduce $\frac{8}{9}$ of a lb Avoirdupois to the fraction of 1 Cwt. *Ans.* $\frac{1}{14}$ Cwt.

C A S E VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

R U L E.

Multiply the given numerator by the parts of the denominations between it and that denomination you would reduce it to, for a new numerator, which place

$$\begin{aligned} * \frac{4}{5} \text{ } \ell &= \frac{4}{5} \text{ of } \frac{20}{1} = \frac{4 \times 20}{5 \times 1} = \frac{80}{5} \text{ s. and } \frac{80}{5} \text{ of } \frac{1}{20} = \frac{80 \times 1}{5 \times 20} \\ &= \frac{80}{100} = \frac{4}{5} \text{ } \text{guin.} \end{aligned}$$

VULGAR FRACTIONS.

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place over the given denominator : Or, only invert the parts contained in the integer, and make of them a compound fraction as before, then, reduce it to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny.

By comparing it, the fraction will be $\frac{1}{400}$ of $\frac{20}{1}$ of

$$\frac{12}{1}, \text{ then } \left\{ \frac{1}{400} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{400} = \frac{3}{5} \text{ answer.} \right.$$

2. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. *Ans.* $\frac{1}{4}qr.$

3. Reduce $\frac{5}{2048}$ of a guinea to the fraction of a penny. *Ans.* $\frac{5}{8}d.$

4. Reduce $\frac{1}{17}$ of a moidore to the fraction of a shilling. *Ans.* $\frac{1}{19}sh.$

5. Reduce $\frac{1}{28}$ of a lb Avoirdupois to the fraction of an ounce. *Ans.* $\frac{2}{7}oz.$

6. *Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound. *Ans.* $\frac{4}{5}l.$

7. Reduce $\frac{7}{1920}$ of a lb Troy to the fraction of a pwt. *Ans.* $\frac{7}{8}pwt.$

8. Reduce $\frac{1}{128}$ of Cwt. to the fraction of a lb Avoirdupois. *Ans.* $\frac{3}{8}lb.$

CASE

$$* \frac{4}{7} \text{ Guin.} = \frac{4}{7} \text{ of } \frac{20}{1} = \frac{4 \times 20}{7 \times 1} = \frac{80}{7}, \text{ \& } \frac{11}{7} \text{ of } \frac{1}{20} = \frac{11}{140} = \frac{1}{12.7}.$$

C A S E IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

R U L E.

Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required.

E X A M P L E S.

1. What is the value of $\frac{5}{7}$ of a pound?

$$\begin{array}{r} 5 \\ 20 \\ \hline 7 \overline{)100} \\ \hline \end{array}$$

$$145.-2$$

$$12$$

$$7 \overline{)24}$$

$$3d.-3$$

$$4$$

$$7 \overline{)12}$$

$$1\frac{5}{7}r.$$

I multiply 5 by 20, the number of shillings in £1, and the product 100 I divide by the denominator 7, and get the quotient 14s. and 2 remaining, I multiply it by 12, and again dividing the product by 7, find the quotient 24d. and 3 remains, which I multiply by 4, and dividing as before, the quotient is 12qr. and 5 remaining, I place it over the numerator, and find the answer 14s. 3d. $1\frac{5}{7}qr.$

2. What

2. What is the value of $\frac{3}{4}$ of a shilling ?

Ans. $4\frac{1}{2}d.$

3. What is the value of $\frac{1}{2}$ of a Cwt. ?

Ans. 2qrs. 9lb 10oz. $7\frac{1}{2}dr.$

4. What is the value of $\frac{4}{5}$ of a lb Avoirdupois ?

Ans. 12oz. $12\frac{2}{3}dr.$

5. What is the value of $\frac{3}{5}$ of a lb Troy ?

Ans. 7oz. 4pwt.

6. What is the value of $\frac{3}{13}$ of a ton ?

Ans. 4Cwt. 2qrs. 12lb 14oz. $12\frac{4}{13}dr.$

7. What is the value of $\frac{3}{5}$ of a yard ?

Ans. 2qrs. $2\frac{2}{3}n.$

8. What is the value of $\frac{1}{4}$ of an ell English ?

Ans. 4qrs. $1\frac{1}{2}n.$

9. What is the value of $\frac{5}{8}$ of a mile ?

Ans. 6fur. 26p. 11ft.

10. What is the value of $\frac{2}{3}$ of a day ?

Ans. 16h. 36m. $55\frac{2}{3}s.$

11. The value of $\frac{2}{14}$ of a guinea is demanded ?

Ans. 18s.

12. What is the value of $\frac{1}{10}$ of a dollar ?

Ans. 5s. $7\frac{1}{2}d.$

13. What is the value of $\frac{3}{5}$ of a moidore ?

Ans. 21s. $7\frac{1}{5}d.$

14. What is the value of $\frac{6}{7}$ of an acre ?

Ans. 3r. $17\frac{1}{7}p.$

CASE

C A S E X.*

To reduce any given quantity to the fraction of any greater denomination of the same kind.

R U L E.

Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term, for a denominator; which will be the fraction required.

E X A M P L E S.

1. Reduce 14s. $3\frac{1}{4}$ d. $\frac{1}{2}$ to the fraction of a pound,
20 integral part.

$$\begin{array}{r} 12 \\ \hline 240 \\ 4 \\ \hline 960 \\ 7 \\ \hline \end{array}$$

6720 denominator.

$$\begin{array}{r} s. \quad d. \\ 14 \quad 3\frac{1}{4}, \frac{1}{2} \\ \hline 12 \\ \hline 17\frac{1}{2} \\ 4 \\ \hline 68\frac{1}{2} \\ 7+5 \\ \hline \end{array}$$

4800 numerator.

$$Ans. \frac{4800}{6720} = \frac{5}{7}.$$

2. Reduce $4\frac{1}{2}$ d. to the fraction of a shilling.

$$Ans. \frac{1}{5}.$$

3. Reduce 2qrs. 9lb 10oz. $7\frac{1}{2}$ dr. to the fraction of
a Cwt.

$$Ans. \frac{1}{25} Cwt.$$

* This case is the reverse of the former, therefore proves it.

NOTE: If there be a fraction given with the said quantity, it must be further reduced to the denominative parts thereof, adding thereto the numerator.

VULGAR FRACTIONS.

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4. Reduce 7oz. 4pwt. to the fraction of a lb Troy.
Ans. $\frac{3}{5}$ lb.
5. Reduce 2grs. $2\frac{2}{3}$ n. to the fraction of a yard.
Ans. $\frac{2}{3}$ yd.
6. Reduce 18s. to the fraction of a guinea.
Ans. $\frac{9}{14}$ G.
7. Reduce 5s. $7\frac{1}{2}$ d. to the fraction of a dollar.
Ans. $\frac{13}{16}$ dol.
8. Reduce 21s. $7\frac{1}{2}$ d. to the fraction of a moidore.
Ans. $\frac{2}{3}$ moid.

ADDITION of VULGAR FRACTIONS.

RULE.

Reduce compound fractions to single ones ; mixed numbers to improper fractions ; fractions of different integers to those of the same ; and all of them to a common denominator ; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add $7\frac{2}{3}$, $\frac{1}{2}$ of $\frac{2}{3}$, and 7 together.

First. $7\frac{2}{3} = \frac{23}{3}$, $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$, and $7 = \frac{7}{1}$.

Then the fractions are $\frac{23}{3}$, $\frac{1}{3}$; and $\frac{7}{1}$; therefore

$$29 \times 56 \times 1 = 2184$$

$$15 \times 5 \times 1 = 75$$

$$7 \times 5 \times 56 = 1960$$

$$\hline 4219$$

$$= 15\frac{19}{280}$$

$$5 \times 56 \times 1 = 280$$

$$\text{Or thus, } \frac{2184 + 75 + 1960}{280} = 15\frac{19}{280}$$

2. What

VULGAR FRACTIONS.

2. What is the sum of $\frac{7}{8}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{2}$ and $9\frac{1}{2}$?

Ans. $12\frac{19}{80}$.

3. Add $\frac{1}{2}l$, $\frac{3}{4}s$, and $\frac{4}{5}d$, together.

Ans. $2s. 8\frac{64}{160}d$.

4. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{3}{4}$ of an hour, and $\frac{3}{4}$ of a minute together.

Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION of VULGAR FRACTIONS.

R U L E.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.—*Note*, a fraction is subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

E X A M P L E S.

1. From $\frac{3}{4}$ take $\frac{2}{3}$ of $\frac{5}{8}$.

$$\begin{array}{l} \frac{2}{3} \text{ of } \frac{5}{8} = \frac{10}{24} = \frac{5}{12}. \text{ Then the fractions are } \frac{3}{4} \text{ and } \frac{5}{12}. \\ 3 \times 28 = 84 \\ 5 \times 4 = 20 \\ 4 \times 28 = 112 \text{ com. den.} \end{array} \left\{ \begin{array}{l} \frac{3}{4} = \frac{84}{112}, \text{ and } \frac{5}{12} = \frac{46\frac{2}{3}}{112}, \\ \text{therefore, } \frac{84}{112} - \frac{46\frac{2}{3}}{112} = \\ \frac{37\frac{1}{3}}{112} = \frac{4}{7} \text{ remainder.} \end{array} \right.$$

2. From

* In subtracting mixed numbers, when the lower fraction (the subtrahend) is greater than the upper one, (the minuend) you may, without reducing them to improper fractions, subtract the numerator of the subtrahend from the common denominator, and to that difference add the numerator of the minuend, and carry one to the integer of the subtrahend.

E X A M P L E.

From $19\frac{3}{4}$ take $12\frac{7}{8}$? — $19\frac{3}{4} - 12\frac{7}{8} = 6\frac{1}{2}$.

VULGAR FRACTIONS.

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2. From $\frac{43}{50}$ take $\frac{5}{9}$. *Ans.* $\frac{171}{450}$.
3. From $37\frac{1}{4}$ take $19\frac{4}{7}$. *Ans.* $17\frac{12}{28}$.
4. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Ans.* $2\frac{1}{12}$.
5. From $\frac{1}{2}$ take $\frac{2}{10}$. *Ans.* $4s. 1\frac{1}{2}d.$
6. From $\frac{5}{7}oz.$ take $\frac{1}{4}pwt.$ *Ans.* $13pwt. 12\frac{5}{7}gr.$

MULTIPLICATION of VULGAR FRACTIONS.

R U L E.

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.—*Note*, where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

E X A M P L E S.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\frac{1}{3} = \frac{13}{3}, \frac{1}{4} \text{ of } \frac{7}{8} = \frac{1 \times 7}{4 \times 8} = \frac{7}{32}, \text{ and } 6 = \frac{6}{1}.$$

$$\text{Then } \frac{13}{3} \times \frac{1}{5} \times \frac{7}{32} \times \frac{6}{1} = \frac{13 \times 1 \times 7 \times 6}{3 \times 5 \times 32 \times 1} = \frac{546}{480} = 1\frac{11}{80} \text{ the answer.}$$

2. Multiply $\frac{3}{17}$ by $\frac{5}{37}$. *Ans.* $\frac{15}{629}$.
3. Multiply $5\frac{1}{2}$ by $\frac{1}{3}$. *Ans.* $\frac{11}{6}$.
4. Multiply $\frac{1}{3}$ of 5 by $\frac{1}{4}$ of $\frac{3}{4}$. *Ans.* $\frac{5}{16}$.

H

DIVISION

DIVISION of VULGAR FRACTIONS.

R U L E.*

Prepare the fractions as before : Then, invert the divisor and proceed exactly as in Multiplication : The products will be the quotient required.

E X A M P L E S.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$.

$\frac{1}{3}$ of 17 = $\frac{1}{3}$ of $\frac{17}{1}$ = $\frac{1 \times 17}{3 \times 1} = \frac{17}{3}$, & $\frac{2}{3}$ of $\frac{6}{8} = \frac{12}{24}$
 = $\frac{1}{2}$; therefore, $\frac{17}{3} \div \frac{1}{2} = \frac{17 \times 2}{3 \times 1} = \frac{34}{3} = 11\frac{1}{3}$ the quotient required.

2. Divide $\frac{5}{7}$ by $\frac{3}{4}$.

Ans. $1\frac{2}{21}$.

3. Divide $12\frac{1}{2}$ by $\frac{1}{3}$ of 7.

Ans. $5\frac{5}{35}$.

4. Divide $5\frac{1}{4}$ by $7\frac{1}{4}$.

Ans. $\frac{41}{82}$.

5. Divide $\frac{3}{7}$ by 9.

Ans. $\frac{1}{31}$.

6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{4}$ of $\frac{2}{4}$.

Ans. $\frac{2}{5}$.

DECIMAL FRACTIONS.

Decimal Fractions are of such a nature, that they vary in the same proportion, and are managed by the same method of operation, as whole numbers are.

On this account, every proper Fraction is supposed to be reducible to another, whose denominator shall be 10, 100, 1000, &c. viz. Unity, with a number of cyphers annexed ; and Fractions with such denominators, are called *Decimal Fractions* : Such are $\frac{5}{10}$, $\frac{55}{100}$, $\frac{575}{1000}$, &c.

As

* To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it ; and to divide by an integer, divide the numerator, or multiply the denominator by it,

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expressed : For the numerator only may be made to express the true value : For this purpose it is only required to write the numerator with a point before it at the left hand, to distinguish it from a whole number, when it consists of so many figures as the denominator hath cyphers annexed to unity, or 1 : So $\frac{5}{10}$ is written ,5 ; $\frac{33}{100}$:33 ; $\frac{735}{1000}$:735, &c.

Note. The point prefixed is called a Separatrix.

But if the numerator has not so many places as the denominator has cyphers, put so many cyphers before it, *viz.* at the left hand, as will make up the defect ; so write $\frac{5}{100}$ thus, ,05 ; and $\frac{6}{1000}$ thus, ,006, &c. And thus do these fractions receive the form of whole numbers.

The 1st, 2^d, 3^d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, seconds, thirds, fourths, &c.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

TABLE.

9	C Millions	9	Tenth Parts
8	X Millions	8	Hundredth Parts
7	Millions	7	Thousandth Parts
6	C Thousands	6	X Thousandth Parts
5	X Thousands	5	C Thousandth Parts
4	Thousands	4	Millionth Parts
3	Hundreds	3	X Millionth Parts
2	Tens	2	C Millionth Parts
1	Units	1	
0		0	

From

From this table it is evident, that, in decimals, as well as in whole numbers, each figure takes its value by its distance from unit's place : If it be in the first place after units (or the separating point) it signifies tenths ; if in the second, hundredths, &c. decreasing in each place in a tenfold proportion.

Consequently, every single figure expressing a decimal, has for its denominator an unit or 1, with so many cyphers as its place is distant from unit's place : Thus, 2 in the decimal part of the table = $\frac{2}{10}$; 3 = $\frac{3}{100}$; 4 = $\frac{4}{1000}$, &c. And if a decimal be expressed by several figures, the denominator is 1, with so many cyphers as the lowest figure is distant from unit's place. So, 357 signifies $\frac{357}{1000}$, and, 0053 = $\frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to possess the same place : So, .5, .50 and .500, are all of the same value, and each equal to $\frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So, .5, .05, .005, all express different decimals, viz. $.5, \frac{5}{10}$; $.05, \frac{5}{100}$; $.005, \frac{5}{1000}$.

Hence may be observed the contrary effect of cyphers being annexed to whole numbers, and decimals.

It is likewise evident from the table, that, since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand toward the left, as the places of whole numbers do : For, ten hundredth parts make one tenth, ten tenths make 1 ; ten units, ten ; ten tens, one hundred, &c. viz. $\frac{10}{100} = \frac{1}{10}$, $\frac{10}{10} = 1$, and $1 \times 10 = 10$, which proves that decimals are subject to the same law of

Notation,

Notation, and consequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of those, which have fewer places, so many cyphers, as make them equal in places with that which has the most. So these decimals, $,5$, $,06$, $,455$, may be reduced to the decimals, $,500$, $,060$, and $,455$, which have, all, 1000 for their denominator.

Of Decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places : Thus, $,5$ is greater than, $,459$, for if it be reduced to the same denominator with $,459$, it will be $,500$.

A mixed number, *viz.* a whole number, with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So $45,309$ is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction :

Thus, $45 \times 1000 + ,309 = \frac{45309}{1000}$ as above.

ADDITION of DECIMALS.

R U L E.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

h

EXAMPLES

DECIMAL FRACTIONS

EXAMPLES.

1. Find the sum of $19,078 + 2,3597 + 223 + ,0197581 + 3478,1 + 12,358$.

$$\begin{array}{r}
 19,078 \\
 2,3597 \\
 223 \\
 ,0197581 \\
 3478,1 \\
 12,358 \\
 \hline
 \end{array}$$

3734,9104581 the sum.

2. Required the sum of $429 + 21,37 + 355,003 + 2,07 + 1,7$? *Ans.* 808,143.

3. Required the sum of $973 + 19 + 1,75 + 93,7161 + ,9501$? *Ans.* 1088,4165.

SUBTRACTION of DECIMALS.

RULE.—Place the numbers according to their value; then subtract as in whole numbers, and point off the decimals as in Addition.

EXAMPLES.

1. Find the difference of 1793,13 and 817,05693 ?

$$\begin{array}{r}
 \text{From } 1793,13 \\
 \text{Take } 817,05693 \\
 \hline
 \end{array}$$

Remainder 976,07307

2. From 171,195, take 125,9176.

Ans. 45,2774

3. From 219,1384 take 195,91.

Ans. 23,2284.

4. From 480 take 245,0875.

Ans. 234,9925.

MULTIPLICATION

DECIMAL FRACTIONS.

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MULTIPLICATION of DECIMALS.

C A S E I.

RULE 1.—Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off so many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect, by prefixing cyphers.

E X A M P L E S,

1. Multiply ,02345
by ,00163

$$\begin{array}{r} 7035 \\ 14070 \\ 2345 \end{array}$$

,0000382235 the product.

2. Multiply 25,238 by 12,17. *Ans.* 307,14646.
3. Multiply ,3759 by ,945. *Ans.* ,3552255.
4. Multiply ,84179 by ,0385. *Ans.* ,032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

$$\text{So ,345} \left\{ \begin{array}{l} \text{Mult. by} \left\{ \begin{array}{l} 10 \\ 100 \\ 1000 \end{array} \right\} \text{ makes} \left\{ \begin{array}{l} 3,45 \\ 34,5 \\ 345 \end{array} \right\} \end{array} \right.$$

For ,345 $\times$ 10 is 3,450, &c.

C A S E II.

To contract the operation, so as to retain so many decimal places in the Product as may be thought necessary.

RULE 1.—Write the unit's place of the multiplier under that figure of the multiplicand, whose place

place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

2. In multiplying, reject all the figures which are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures, when you begin to multiply, and the sum will be the product required.

E X A M P L E S.

1. It is required to multiply 56,7534916 by 5,376928, and to retain only five places of decimals in the product.

Contracted way.

56,7534916
829673,5

28376746 . .
1702605 . . .
397274
34052
5108
113
45

305,15943

Common way.

56,7534916
5,376928

45	40279328
113	5069832
5107	814344
84052	99496
397274	4412
1702604	748
28376745	80

305,15943|80818048

By the operation in the *common way*, it is evident that all the figures, which are cut off at the right hand, by the perpendicular line, are wholly omitted in the *contracted way*, and the last product here is the first there; consequently the reason of placing the multiplier in a reverse order, must appear very plainly.

DIVISION

DECIMAL FRACTIONS.

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DIVISION of DECIMALS.

RULE 1.—The places of decimal parts in the divisor and quotient counted together, must always be equal to those in the dividend, therefore divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

EXAMPLES.

1.	2.
$\begin{array}{r} 119)117841075 \\ \underline{1095} \\ 834 \\ \underline{657} \\ 1771 \\ \underline{1752} \\ 1907 \\ \underline{1752} \\ 1555 \\ \underline{1533} \\ 22 \end{array}$	$\begin{array}{r} 3719)33,0000(102,172, \&c. \\ \underline{3719} \\ 8100 \\ \underline{7438} \\ 6620 \\ \underline{3719} \\ 29010 \\ \underline{26033} \\ 29770 \\ \underline{29752} \\ 18 \end{array}$
In Example 1st, the divisor having no decimals, the quotient must have so many as there are in the dividend.	
In Example 2, the dividend being an integer must have at least so many cyphers annexed as there are decimals in the divisor, & so far the quotient will be whole numbers, then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.	
3d. $133)5737(43,1353+$	
4th. $23,7)65321(2756,16+$	
5th. $*72)918,217(12753+$	
	6th.

* The following questions are left unsolved in the quotient, to exercise the learner.

$$\begin{array}{l}
 6\text{th. } 25,17)315,693(1253+ \\
 7\text{th. } ,317)29,417(92+ \\
 8\text{th. } 37,9),0059374(156+ \\
 9\text{th. } ,375),173948375(463862+
 \end{array}$$

C A S E II.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE 1. Whatever place of the dividend corresponds with the unit's place of the divisor, at the first step of the division, the same place must the first figure of the quotient have.

2. In dividing, reject the last right hand figure of the divisor, at every step (instead of bringing down a figure, as in common) and make the last remainder the dividend for the new divisor at every step:—Thus continue the division until the divisor shall be exhausted.

$$\begin{array}{r}
 \text{.....} \\
 99,5678)4,6789837568(,0469931 \text{ Quotient.} \\
 \underline{3982712}
 \end{array}$$

$$\begin{array}{r}
 99,567)696271 \\
 \underline{597402}
 \end{array}$$

$$\begin{array}{r}
 99,56)98869 \\
 \underline{89604}
 \end{array}$$

$$\begin{array}{r}
 99,5)9265 \\
 \underline{8955}
 \end{array}$$

$$\begin{array}{r}
 99)810 \\
 \underline{297}
 \end{array}$$

$$\begin{array}{r}
 9)13 \\
 \underline{9}
 \end{array}$$

$$\begin{array}{r}
 \text{Remainder } 4
 \end{array}$$

Here, the unit's place of the divisor in the first step falls under 7 in the place of hundredths in the dividend, therefore I put 4, the first quotient figure, in the place of hundredths, by prefixing a cypher.

I have set down every divisor to explain the work; but you need only put a dash over every figure rejected, as you proceed, to shew it is omitted.

When

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers) it is performed by removing the separatrix, in the dividend, so many places toward the left and as there are cyphers in the divisor.

EXAMPLES.

$$\begin{array}{r} 10 \\ 100 \\ 1000 \\ 10000 \end{array} \left. \begin{array}{l} \text{Dividing} \\ \left\{ \begin{array}{l} 7654 \end{array} \right\} \end{array} \right\} \begin{array}{l} \text{The Quotient is} \\ \left\{ \begin{array}{l} 765,4 \\ 76,54 \\ 7,654 \\ ,7654- \end{array} \right\} \end{array}$$

REDUCTION of DECIMALS.

CASE I.

To reduce a Vulgar Fraction to its equivalent Decimal.

RULE. Divide the numerator by the denominator, as in division of decimals, and the quotient will be the decimal required:—Or, so many cyphers as you annex to the given numerator, so many places must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many cyphers before the quotient figures.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal. $\begin{array}{r} 8 \overline{)1,000} \\ \underline{125} \end{array}$ *Ans.*

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{3}$ to decimals. *Answers.* ,375. ,625. ,666+

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$ to decimals. *Answers.* ,25. ,5. ,75. ,333+. ,8. ,833+. ,875.

4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{80}$ and $\frac{9}{36}$ to decimals. *Answers.* ,263+. ,692+. ,025. ,25.

CASE

C A S E II.

To reduce numbers of different denominations, as of Money, Weight and Measure, to their equivalent decimal values.

RULE 1. Write the given numbers perpendicularly, under each other, for dividends; proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number, for a divisor, as will bring it to the next superior denomination, and draw a line perpendicularly between them.

3. Begin with the highest, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used, and the last quotient will be the decimal sought.

1. Reduce 17s. 8½d. to the decimal of a pound.

$$\begin{array}{r|l}
 4 & 3, \\
 12 & 8,75 \\
 \hline
 20 & 17,729166, \text{ \&c.}
 \end{array}$$

,886458, &c. the decimal required.

Here, in dividing 3 by 4, I suppose 2 cyphers to be annexed to the 3, which make it 3,00, and ,75 is the quotient, which I write against 8 in the next line; this quotient, viz. 8,75 being pence and decimal parts of a penny, I divide them by 12, which brings them to shillings and decimal parts, I therefore divide by 20, and (there being no whole number) the quotient is decimal parts of a pound.

2. Reduce 1, 2, 3, 4, and so on, to 19 shillings, to decimals.

Shillings.	1	2	3	4	5	6
Answers.	,05.	,1.	,15.	,2.	,25.	,3.
Shillings.	7	8	9	10	11	12
Answers.	,35.	,4.	,45.	,5.	,55.	,6.

Shillings,

Shillings. 13 14 15 16 17 18 19
Answers. ,65. ,7. ,75. ,8. ,85. ,9. ,95.

Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cypher to the shillings, and then halving them, you will have their decimal expression.

3. *Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.

Pence. 1 2 3 4 5 6
Answers. ,083+. ,166. ,25. ,333+. ,416+. ,5.
 Pence. 7 8 9 10 11
Answers. ,583+. ,666+. ,75. ,833+. ,916+.

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence. 1 2 3 4
Answers. ,00416+. ,0083+. ,0125. ,01666+.
 Pence. 5 6 7 8
Answers. ,0208+. ,025. ,03916+. ,0833+.
 Pence. 9 10
Answers. ,0875. ,0416+. ,04583+.

5. Reduce 1, 2 and 3 farthings to the decimals of a penny.

1qr. = ,25d. 2qr. = ,5d. and 3qr. = ,75d. *Answers.*

6. Reduce 1, 2 and 3 farthings to the decimals of a shilling.

Answers. 1qr. = ,02083+s. 2qrs. = ,04166+s.
 3qrs. = ,0625s.

7. Reduce 1, 2 and 3 farthings to the decimals of a pound.

Ans. 1qr. = ,0010416+£. 2qrs. = ,002083+£.
 3qrs. = ,003125£.

8. Reduce 13s. 5½d. to the decimal of a pound.

Ans. ,6729+.

9. Reduce 7Cwt. 3qr. 17lb 10oz. 12dr. to the decimal of a ton.

Ans. ,39538+.

10. Reduce

* The answers to this question are the same as the decimal parts of a foot.

10. Reduce 1002. 13*wt.* 9*gr.* to the decimal of a pound Troy. *Ans.* .8890625.

11. Reduce 3*grs.* 3*n.* to the decimal of a yard. *Ans.* .9375.

C A S E III.

To find the decimal of any number of shillings, pence and farthings, by Inspection.

RULE 1.—Write *half* the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third places ; observing, to increase the *second* place or place of *hundredths*, by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

3. If more than three places be needful, divide half the number of farthings in the given pence and farthings (rejecting 24*grs.* first, if there be so many) by 12, and the quotient, written after the three places before found, will give the decimal required.

E X A M P L E S.

1. Find the decimal of 13*s.* 9½*d.* by inspection.

6 . . . = ½ of 12*s.*

5 for the odd shilling.

39 = the farthings in 9½*d.*

Add 2 for the excess of 36.

—
,691 = decimal required.

2. Find, by Inspection, the decimal expressions of 18*s.* 3½*d.* and 17*s.* 8½*d.*

Ans. £.914 and £.885.

3. Value the following sums, by Inspection, and find their total, viz. 15*s.* 3*d.* + 8*s.* 11½*d.* + 10*s.* 6½*d.* + 1*s.* 8½*d.* + ½*d.* + 2½*d.* *Ans.* £.1,834 the total.

CASE

C A S E IV.

To find the value of any given decimal in the terms of the Integer.

RULE 1.—Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before.

3. Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer.

E X A M P L E S.

1. Find the value of .73968 of a pound.

$$\begin{array}{r}
 20 \\
 \hline
 14,79360 \\
 12 \\
 \hline
 9,32320 \\
 4 \\
 \hline
 2,09280
 \end{array}$$

Ans. 14s. 9½d.

2. What is the value of .679 of a shilling?

Ans. 8,148d.

3. What is the value of .9999£.?

Ans. 19s 11½d. ½—or £.1.

4. What is the value of .617 of a Cwt.?

Ans. 2qrs. 13lb. 10z. 10⅙dr.

5. What is the value of .8593 of a lb Troy?

Ans. 100z. 62wt. 5gr.

6. What is the value of .297 of a yard?

Ans. 1qr. 2n.

7. What

7. What is the value of ,8469 of a degree ?

Ans. 58m. 6fur. 35po. 0ft. 11in.

8. What is the value of ,569 of a year ?

Ans. 207da. 16h. 26m. 24sec.

C A S E V.

To find the value of any decimal of a pound by inspection.

RULE.—Double the first figure, or place of tenths, for shillings, and if the second figure be 5, or more than 5, reckon another shilling; then, after the 5 is deducted, call the figures in the second and third places so many farthings, abating 1 when they are above 12, and 2 when above 36, and the result will be the answer.

Note. When the Decimal has but 2 figures, if any thing remain after the shillings are taken out, a cypher must be annexed to the right hand, or supposed to be so.

E X A M P L E S.

1. Find the value of ,876£. by Inspection.

16s. = double of 8.

1s. for the 5 in the second place, which
(is to be taken out of 7,

and 6½d. = 26 farthings, remain to be added.
deduct ¼d. for the excess of 12.

17s. 6½d. the *Ans.*

2. Find, by Inspection, the value of ,49£.

8s. - - = double of 4.

1s. - - for the 5 in the place of hundredths.

10d. = 40 farthings, a 0 being annexed to
(the remaining 4.

Ded. ¼d. for the excess of 36.

9s. 9½d. the answer.

3. Find

FEDERAL MONEY.

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3. Find the value of ,097£. by Inspection.

Ans. 1s. 11½d.

4. Value the following decimals, by Inspection, and find their sum, viz. ,785£. + ,537£. + ,916£. + ,74£. + ,5£. + ,25£. + ,09£. + ,008£.

Ans. £ 3 16s. 6d.

FEDERAL MONEY.

The pupil being well acquainted with decimals, it will be proper to introduce here an account of the Federal Money.

As the Money of Account proceeds in a decuple, or tenfold proportion, so, any number of Dollars, Dimes, Cents and Mills, is, simply the expression of Dollars and Decimal parts of a Dollar:—Thus 9 Dollars and 8 Dimes are expressed 9,8 = $9\frac{8}{10}$ doll.—12 Dollars, 4 Dimes, and 7 Cents, thus, 12,47 = $12\frac{47}{100}$ doll. 20 Dollars, 3 Dimes, 4 Cents and 5 Mills, thus, 20,345 = $20\frac{345}{1000}$ doll. 100 Dollars and 9 Mills, thus, 100,009 = $100\frac{9}{1000}$ doll. And 50 Dollars, 5 Cents, thus, 50,05 = $50\frac{5}{100}$ doll. Wherefore it is, in all respects, added, subtracted, multiplied and divided, the same as Decimals; and, of all Coins, it is the most simple.

		marked,	
Note	10 Mills	make one	Cent. m. c.
	10 Cents		Dime. d.
	10 Dimes		Dollar. D.
	10 Dollars		Eagle. E.

ADDITION of the FEDERAL MONEY.

Add 25½ Eagles; 7 Dollars, 8 Dimes, 3 Cents, 4 Mills; 125 Dollars, 8 Cents; 5 Eagles, 9 Mills; 18 Dollars, 7 Cents and 4 Mills, together.*

Ans.

* It may be observed, that the sum exhibits the particular number of each different piece of money contained in it, viz. 455997 Mills = $45599\frac{7}{100}$ Cents = $4559\frac{97}{100}$ Dimes = $455\frac{997}{1000}$ Dol-

E. D. d. c. m.

lars = $45\frac{997}{10000}$ Eagles = 45. 5. 9 9 7.

Also,

FEDERAL MONEY.

	Eagles.	Dollars.	Dimes.	Cents.	Mills.	
1st.	25	5				
2d.		7	8	3	4	
3d.	12	5	0	8	0	
4th.		5	0	0	9	
5th.		1	8	0	7	
Sum	45	5	9	9	7	

Note, That Dollars occupy the first place at the left hand of the comma, and Eagles, all the places at the left of Dollars: But Eagles and Dollars, reckoned together, express the number of Dollars contained in the sum, as 349 is 34 Eagles and 9 Dollars; equal to 349 Dollars, &c.

SUBTRACTION.

	E. D. d. c. m.	D. d. c. m.	D. d. c. m.
From	13479,8 1 5	495641,0 0 1	798,
Take	8985,9 4 6	218796,7 9 5	459,8 7 9
Rem.	4493,8 6 9	276844,2 0 6	338,6 2 1
Proof.	13479,8 1 5		

MULTIPLICATION.

When you have pointed off the decimals in the product, according to the rule in Multiplication of Decimals, all beyond the Mills, or third place of decimals, are decimal parts of a Mill.

Bought 27 horses for shipping, at $D. d. c. m.$ 48, 5 7 3 per head: What came they to?

$D. d. c. m.$

Also, the names of the Coins, less than a Dollar, are significant of their values.

For the *Mill*, which stands in the 3d place, at the right hand of the comma, or place of thousandths, is contracted from *Mille*, the Latin for *Thousand*:—*Cent*, which occupies the second place, or place of Hundredths, is an abbreviation of *Centum*, the Latin for *Hundred*:—And *Dime*, which is in the first place, or place of tenths, is derived from *Disme* the French for *Tenths*.

FEDERAL MONEY. 109

$$\begin{array}{r}
 D. d. c. m. \\
 48,573 \\
 87 \\
 \hline
 340011 \\
 145719 \\
 \hline
 \text{Ans. } 1797,201 \\
 D. d. c. m.
 \end{array}$$

$$\begin{array}{r}
 D. d. c. m. \\
 \text{Mult. } 45,846 \\
 \text{by } 23,094 \\
 \hline
 183384 \\
 412614 \\
 \hline
 1375380 \\
 91692 \\
 \hline
 \text{Ans. } 1058,767 \frac{524}{1000} \\
 D. d. c. m.
 \end{array}$$

DIVISION.

1. If 1000 oranges cost 10 dollars, what is that a piece?

$$\begin{array}{r}
 D. d. c. D. d. c. \\
 1000 \overline{) 0,00(0,01} \text{ Ans.} \\
 1000 \\
 \hline
 \end{array}$$

2. If 3730 bushels of corn cost 2025,39, what is that per bushel?

$$\begin{array}{r}
 D. d. c. D. d. c. m. \\
 3730 \overline{) 2025,39(0,543} \text{ Ans.} \\
 18650 \\
 \hline
 \end{array}$$

16039

14940

11190

11490

3. Divide 12976 dollars between 7 men.

$$\begin{array}{r}
 7 \overline{) 12976} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1853,714 \frac{2}{7} \text{ Ans. in dol. dim. \&c.} \\
 D. d. c. m.
 \end{array}$$

DECIMAL

* And here I would remind the learner, that, when he has brought down all his whole numbers, or dollars, in the dividend, he must place a comma in the quotient; and if, when he has brought down the next figure, he cannot have the divisor once, he must place a cypher at the right hand of the comma, in the place of dimes.

DECIMAL TABLES of Coin, Weight and Measure.

TABLE I. COIN.
£ 1 the Integer.

Shil.	dec.	Shil.	dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		,0

Pence.	Decimals.
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farthings.	Decimals.
3	,003125
2	,0020833
1	,0010416

TABLE II.
Coin and Long Meas.
1 Shilling & 1 Foot
the Integer.

Pence & Inches.	Decimals.
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	,083333

Farthings.	Decimals.
3	,0625
2	,041666
1	,020833

TABLE III.
TROY WEIGHT.
1 lb the Integer.
Ounces the same as

Penny-weights.	Decimals.
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains.	Decimals.
12	,002083
11	,00191
10	,001736
9	,001562
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

1 Oz. the Integer.
Pennyweights the same
as Shillings in the first
Table.

Grains.	Decimals.
12	,025
11	,022916
10	,020833
9	,01875
8	,016666
7	,014583
6	,0125

Grains.	Decimals.
5	,010416
4	,008333
3	,00625
2	,004166
1	,002083

TABLE IV.
AVOIRDUPOIS WT.
1 lb the Integer.

Qrs.	Decimals.
3	,75
2	,5
1	,25

Pounds.	Decimals.
27	,241071
26	,232143
25	,223214
24	,214286
23	,205357
22	,196428
21	,1875
20	,178571
19	,169643
18	,160714
17	,151786
16	,142857
15	,133928
14	,125
13	,110761
12	,107143
11	,098214
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces	Decimals
16	,008370
14	,007812
13	,007254
12	,006696
Carried over.	

DECIMAL TABLES of Weight and Measure.

Brought over.		Drams.	Decimals.	Yards.	Decimals.	
Ounces.	Decimals.	7	,027343	7	,003977	
11	,006138	6	,023437	6	,003409	
10	,00558	5	,019531	5	,002841	
9	,005022	4	,015625	4	,002273	
8	,004464	3	,011718	3	,001704	
7	,003906	2	,007812	2	,001136	
6	,003348	1	,003906	1	,000568	
5	,00279	TABLE VII. CLOTH MEASURE. 1 Yard the Inte- ger.			Feet.	Decimals.
4	,002232				2	,0003787
3	,001674				1	,0001892
2	,001116				Inches.	Decimals.
1	,000558				6	,0000947
					5	,000079

qrs. of ozs.	Decim.	Quarters.	Decimals.
3	,000418	3	,75
2	,000279	2	,5
1	,000139	1	,25

TABLE V.
Avoir. Weight.
1 lb the Integer.

Ounces.	Decimals.
15	,9375
14	,875
13	,8125
12	,75
11	,6875
10	,625
9	,5625
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Nails.	Decimals.
3	,1875
2	,125
1	,0625

TABLE VI.
LONG MEASURE.
1 Mile the Integer.

Yards.	Decimals.
1000	,568182
900	,511864
800	,454545
700	,397227
600	,34
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051186
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545

TABLE VIII.
LIQUID MEASURE.
1 Gallon the Integer.
Quarts the same as
qrs. in TABLE VI.

1 Pint.	,125
3 Gills.	,09375
2	,0625
1	,03125

TABLE IX.
TIME.
1 Year the Integer.
Months the same as
Pence in TABLE II.

Days.	Decimals.
365	1,000000
300	,821928
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
Carried over.	

Drams.	Decimals.
15	,059493
14	,055587
13	,051681
12	,047775
11	,043868
10	,039962
9	,036056
8	,03215

166 COMPOUND MULTIPLICATION.

Brought over.		Hours.	Decimals.	Minutes.	Decimals.
Days.	Decimals.	19	,791666	30	,020833
30	,082192	18	,75	20	,013888
20	,054794	17	,708333	10	,006944
10	,027397	16	,666666	9	,00625
9	,024657	15	,625	8	,005555
8	,021918	14	,583333	7	,004861
7	,019178	13	,541666	6	,004166
6	,016438	12	,5	5	,003472
5	,013698	11	,458333	4	,002777
4	,010959	10	,416666	3	,002083
3	,008219	9	,375	2	,001388
2	,005479	8	,333333	1	,000694
1	,002739	7	,291666		
1 Day the Integer.		6	,25		
		5	,208333		
Hours.	Decimals.	4	,166666		
23	,958333	3	,125		
22	,916666	2	,083333		
21	,875	1	,041666		
20	,833333				

COMPOUND MULTIPLICATION

Is extremely useful in finding the value of Goods, &c.—And, as in Compound Addition, we carry from the lowest denomination to the next higher, so, we begin and carry in Compound Multiplication; one general rule being, to multiply the price by the quantity.

C A S E I.

When the quantity does not exceed 12 yards, pounds, &c. set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for carrying from one denomination to another, as in Compound Addition.

INTRODUCTION

COMPOUND MULTIPLICATION. 107

INTRODUCTORY EXAMPLES.

	1.		2.		3.
	£. s. d.	£. s. d.	£. s. d.		
Multiply	15 17 1	25 12 8	45 9 10		
by	2	8	4		
	_____	_____	_____		
	_____	_____	_____		
	67 18 6½	19 9 8½	13 12 11		
	5	6	7		
	_____	_____	_____		
	_____	_____	_____		
	31 16 8½	12 17 10½			
	8	9			
	_____	_____			
	_____	_____			

PRACTICAL QUESTIONS.

1. What will 9 yards of cloth, at 5s. 4d. per yard, come to?

£. s. d.
 0 5 4 price of one yard,
 Multiplied by 9 yards.

Ans. £. 2 8 0 price of 9 yards.

Questions.

2d. 3 yards, at 15 4 }
 3d. 6 — at 9 10 } per yard.
 4th. 9 — at 13 7½ }

CASE II.

When the multiplier, that is, the quantity, is above 12, you must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

EXAMPLES.

1. What will 144 yards of cloth cost, at 9s. 5½d. per yard.

108 COMPOUND MULTIPLICATION.

$\begin{array}{r} \text{£. s. d.} \\ 0 \ 3 \ 5\frac{1}{2} \text{ price of one yard,} \\ \text{Multiplied by} \quad 12 \\ \hline \text{Produces} \quad 2 \ 1 \ 6 \text{ price of 12 yards,} \\ \text{Multiplied by} \quad 12 \\ \hline \end{array}$

Ans. £. 24 18 0 price of 144 yards

Questions.	s.	d.	Answers.	£.	s.	d.
2d. 24 yards at 6 3 $\frac{1}{2}$ peryard.	6	3 $\frac{1}{2}$	=	7	11	6
3d. 27 — at 9 10	9	10	=	13	5	6
4th. 44 — at 12 4 $\frac{1}{2}$	12	4 $\frac{1}{2}$	=	27	4	6

C A S E III.

When the quantity is such a number, as that no two numbers in the table will produce it exactly; Then Multiply by two such numbers as come the nearest to it; and for the number wanting, multiply the given price of one yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

E X A M P L E S.

1. What will 47 yards of cloth, at 17s. 9d. per yard, come to?

$\begin{array}{r} \text{£. s. d.} \\ 0 \ 17 \ 9 \text{ price of one yard.} \\ \text{Multiplied by} \quad 5 \\ \hline \text{Produces} \quad 4 \ 8 \ 9 \text{ price of 5 yards.} \\ \text{Multiplied by} \quad 9 \\ \hline \text{Produces} \quad 39 \ 18 \ 9 \text{ price of 45 yards.} \\ \text{Add} \quad 1 \ 15 \ 6 \text{ price of 2 yards.} \\ \hline \text{Ans.} \quad 41 \ 14 \ 3 \text{ price of 47 yards.} \end{array}$

Note.

COMPOUND MULTIPLICATION. 109

Note.—This may be performed by first finding the value of 48 yards, from which, if you subtract the price of 1, the remainder will be the answer, as above.

Questions.

Yds. s. d.

1st. 75 at 5 7½ per yard, =
2d. 67½ — 16 3½ — =
3th. 59 — 9 7 — =

Answers.

£. s. d.

21 1 10½
54 18 3½
28 5 5

C A S E IV.

When the quantity is any number above the Multiplication Table; Multiply the price of 1 yard by 10, which will produce the price of 10 yards: This product, multiplied by 10, will give the price of 100 yards; then, if the quantity do not exceed hundreds, you must multiply the price of one hundred by the number of hundreds in your question; the price of ten by the number of tens; and the price of unity, or 1, by the number of units: Lastly, add these several products together, and the sum will be the answer.

E X A M P L E S.

1. What will 359 yards of cloth, at 4s. 7½d. per yard amount to?

£. s. d.

0 4 7½ price of one yard.

2 6 3 price of 10 yards.

23 2 6 price of 100 yards.

69 7 6 price of 300 yards.

11 11 3 price of 50 yards.

2 11 7½ price of 9 yards.

Ans. £83 0 4½ price of 359 yards.

Questions.

110 COMPOUND MULTIPLICATION.

Questions.	s.	d.	£.	s.	d.
2d. 297 yards, at 17 3½ per yard, =	256	15 7½			
3d. 473 ————— 9 11½ ————— =	235	0 5½			
4th. 512 ————— 15 10 ————— =	405	0 0			

C A S E V.

When the quantity does not exceed 200, nor the price 12 pence, then, by the pence table, find what it comes to, at one penny per yard, &c. and multiplying this sum by the number of pence in the price, the product will be the answer.

E X A M P L E S.

1. What will 120 yards cost, at 9½d. per yard?
- | | | |
|----------------------------|----|----|
| £. | s. | d. |
| 10 | 10 | 9 |
| the price at 1d. per yard. | | |
| 4 | 16 | 9 |
| the price at 9d. per yard. | | |
| Half of 10s. 9d. = | 5 | 4½ |
| the price at ½d. per yard. | | |
| a fourth of 10/9 = | 2 | 8½ |
| the price at ¼d. per yard. | | |
- Ans. £5 4 9½ the price at 9½d. per yard.

Questions.

Yds.

Answers.

	£.	s.	d.
2d. 148½ at 11½d. per yard, =	6	19	2½
3d. 75 — 7 ————— =	2	3	9
4th. 63½ — 3½ ————— =	0	19	10

C A S E VI.

To find the value of one hundred weight :—As 112 is the gross hundred, so 112 farthings are = 2s. 4d. and 112 pence = 9s. 4d. therefore if the price be farthings, or not more than 3d multiply 2s. 4d. by the farthings in the price of 1lb, or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1lb, and in either case the product will be the answer.

E X A M P L E S.

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EXAMPLES.

1. What will 1 Cwt. of chalk come to, at $1\frac{1}{2}d.$ per lb?

$112 \text{ farthings} = 28 \text{ s.}$
 $112 \text{ farthings} = 28 \text{ s.}$
 $112 \text{ farthings} = 28 \text{ s.}$

Ans. £0 14 0 price of 1 Cwt. at $1\frac{1}{2}$ d. per lb.

2. 1 Cwt. of tin at $2\frac{1}{2}d.$ per lb.

	s.	d.
Price of 1 Cwt. at $\frac{1}{4}d.$ per lb,	2	4
Farthings in the price of 1 lb,		9

Price of 1 Cwt. at $2\frac{1}{2}d.$ per lb. Ans. £ 1 1 0

g. 1 Cwt. of lead at 7d. per lb.

	3	2	1	0	s.	d.
Price of 1 Cwt. at 1d. per lb.					3	4
Pence in the price of 1 lb.						7

Price of 1 Gwt. at 7d. per lb. £ 3 5 4

Questions.

Crew

Answers.

Cash £. s. d.

4th. 1 of Copper at $\frac{1}{2}d.$ per lb. $\equiv 0 \ 7 \ 0$

grh. 1 ditto ——— 2 d. 1 q. ——— 1 4 6

6th. 2 ditto	4	2	2	0
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CASE VII.

To find the value of two, or more hundreds, by having the price of one pound :—First, find the price of 1 Cwt. by the last Case, and then proceed to find the value of the whole, by Case 1st, or 2d, as the question may require.

EXAMPLES.

1. What is the value of $5\frac{1}{2}$ Cwt. of sugar, at 6d. per lb?

f. s. d.

112 COMPOUND MULTIPLICATION.

$$\begin{array}{r} \text{£. s. d.} \\ 0 \quad 9 \quad 4 \text{ price of 1 Cwt. at 1d. per lb.} \\ \hline \end{array}$$

$$\begin{array}{r} 2 \quad 16 \quad 0 \text{ price of ditto at 6d. per lb.} \\ \hline 5 \end{array}$$

$$\begin{array}{r} 14 \quad 0 \quad 0 \text{ price of 5 Cwt.} \\ \text{gr. of Cwt.} = 0 \quad 14 \quad 0 \text{ price of } \frac{1}{4} \text{ Cwt.} \\ \hline \end{array}$$

Answer, £14 14 0 price of 5¼ Cwt.

Questions.

Answers.

Cwt.	ad.	4	of sugar at 3½d. per lb. =	£.	s.	d.
	3d.	8½	5d. =	4	13	4
	4th.	7	4½d. =	19	16	8
				15	10	4

C A S E VIII.

To find the value of a hundred weight when the price of 1 lb is any number of pounds and shillings; or shillings, pence and farthings:—Multiply the price of 1 lb by 7, its product by 8, and this product by 2; which last product will be the answer required.

E X A M P L E S.

1. What will 1 Cwt. of Tobacco cost, at 5s. 7½d. per lb?

$$\begin{array}{r} \text{£. s. d.} \\ 0 \quad 5 \quad 7\frac{1}{2} \text{ price of 1 lb.} \\ \hline \end{array}$$

$$\begin{array}{r} 1 \quad 19 \quad 2\frac{1}{2} \text{ price of 7 lb.} \\ \hline \end{array}$$

$$\begin{array}{r} 15 \quad 15 \quad 0 \text{ price of 56 lb, or } \frac{7}{4} \text{ Cwt.} \\ \hline \end{array}$$

Ans. £31 10 0 price of 112 lb, or 1 Cwt.

Questions.

COMPOUND DIVISION.

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Questions.

Answers.

			s.	d.			£.	s.	d.
1st.	1	Cwt. at	3	10 ^s	per lb.	=	21	14	0
2d.	1	ditto—	9	6	—	=	53	4	0
4th.	1	ditto—	16	11 ¹ / ₂	—	=	94	10	4

COMPOUND DIVISION

Is the dividing of numbers of different denominations: In doing which, always begin at the highest, and when you have divided that, if any thing remains, reduce it to the next lower denomination, and so on, till you have divided the whole; taking care to set down your quotient figures under their respective denominations.

INTRODUCTORY EXAMPLES.

	£.	s.	d.		£.	s.	d.
Divide	549	17	9	by 5	4)731	5	10 ^s
Quotient	£.	109	19	6 ^s			

£.	s.	d.	£.	s.	d.	£.	s.	d.
2)97	19	10 ^s	6)37	13	4 ^s	7)193	15	9 ^s

6.					7.				
T. cwt. qr. lb. oz. dr.					lb. oz. pwt. gr.				
3	29	13	2	25, 12 13	12	5	19	10	7 14

8.							9.						
Deg.	m.	f.	p.	sq.	in.	br.	lb.	3	3	9	gr.		
6	97	55	7	35	4	2	1	3	7	10	5	1	12

10. Suppose

10. Suppose that two places lie east and west of each other, and it is found by observation, that it is noon at the former 2 hours, 6' 30" sooner than at the latter; how many degrees are they asunder?

4) 2h. 6' 30" Reduce the hours and minutes
to minutes, then, minutes divided
31° 37' 30" *Ans.* by 4, give degrees in the quotient.

11. The longitude of Cambridge is 4h. 44' 17", and that of Philadelphia, 5h. 0' 43"; how many degrees difference?

5h. 0' 43"
4h. 44' 17"
4) 0 16 26
4h. 6' 30" *Ans.*

PRACTICAL QUESTIONS.

C A S E I.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 yard:— If the quantity do not exceed 12, proceed by setting down the price and dividing it by the quantity; which quotient will be the price of 1 yard, required; but if the quantity exceed 12, then divide by 2 such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Note. This case proves the first and second cases in Compound Multiplication.

1. If 9 yards of cloth cost £4 3s. 7½d. what is it per yard?

£. s. d.
9) 4 3 7½
0 9 8½ *Ans.*

COMPOUND DIVISION.

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2. If 7 ells cost £5 17s. 5d. what cost 1 ell?
3. If 11 sheep cost £6 5s. 9d. what did each cost?
4. If 12 gallons of rum cost £8 11s. 9½d. what is it per gallon?

Note. When the given quantity (or divisor) is large, and not a composite number, the operation may be performed by long Division.

C A S E II.

Having the price of a hundred weight, to find the price of 1lb.—Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1lb. required.

1. If 1 Cwt. of flax cost £2 7s. 8d. what is that per lb?

$$\begin{array}{r} \text{£. s. d.} \\ 8 \overline{) 2 \ 7 \ 8} \end{array}$$

$$\begin{array}{r} 7 \overline{) 0 \ 5 \ 11} \end{array}$$

$$\begin{array}{r} 2 \overline{) 0 \ 0 \ 10 \ 0} \end{array}$$

= 0 5½d. price of one pound.

2. At £3 10s. per Cwt. what cost 1lb? *Ans.* 7½d.
3. At £6 6s. per Cwt. what cost 1lb? *Ans.* 1s. 1½d.
4. At £42 11s. 8d. per Cwt. what cost 1lb? *Ans.* 7s. 7½d.

C A S E III.

Having the price of several hundred weight, to find the price per lb.—Divide the whole price by the number of hundreds, which will give the price per Cwt. and then proceed as in the last Case.

1. If 5 Cwt. of sugar cost £12 8s. 4d. what is that per lb?

£.

COMPOUND DIVISION

$$\begin{array}{r} \text{£. s. d.} \\ 5) 13 \text{ } 8 \text{ } 4 \end{array}$$

8) 2 13 8 price of 1 Cwt.

2) 0 6 8 price of 14 lb. or $\frac{1}{2}$ Cwt.

2) 0 0 11 price of 2 lb. or $\frac{1}{8}$ Cwt.

0 0 5 price of 1 lb.

2. If 8 Cwt. of cocoa cost £15 17s. 4d. what is that per lb.?

3. If $3\frac{1}{4}$ Cwt. of sugar cost £9 17s. 2d. what is that per lb.?

4. If $1\frac{1}{4}$ Cwt. of cotton wool cost £6 10s. 8d. what is that per lb.?

Note. This Case proves the 7th. in Compound Multiplication.

G. A. S. E. LV.

Having the price of any number of yards, &c. to find the price of 1 yard:—Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferior denomination, and, at each reduction, divide as before, remembering, each time, to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed.

1. If $95\frac{1}{4}$ lb. of figs cost £16 13s. 6d. what are they per lb.?

Quantity = $95\frac{1}{4}$ Price = £16 13s. 6d.

Mult. by 4

Produces 382 for a div. Pro. £66 14s. 3 for a div.

£.

REDUCTION OF COINS.

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$$\begin{array}{r} \text{£. s. d.} \quad \text{£. s. d.} \\ 382)66 \ 14 \ 8 \ (0 \ 3 \ 5\frac{1}{2} \ \frac{112}{112} \text{ per } \text{lb.} \\ 20 \end{array}$$

$$\begin{array}{r} 382)1334(3 \\ 1146 \\ \hline 188 \\ 18 \\ \hline \end{array}$$

$$\begin{array}{r} 382)2259(5 \\ 1910 \\ \hline 349 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 382)1396(3 \\ 1146 \\ \hline 250 \end{array}$$

2. If 147 bushels of rye cost £47 12s. 6d. what is it per bushel ?

Ans. 6s. 5½d.

3. If 33½ yards of baize cost £25 13s. 9½d. what is it per yard ?

Ans. 25s. 5½d. ½d.

Note. This proves the 3d and 4th Cases in Multiplication.

R U L E S

For reducing the Federal Coin, and the Currencies of the several United States ; also English, Irish, Canada, Nova Scotia, Livres Tournois, and Spanish milled Dollars, each, to the *par* of all the others.

1. To reduce New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency, to decimals, by Inspection, (Case 3d, Dec. Frac.) divide the whole by 3, putting the comma one figure further to the right hand in the quotient, than in the pounds of
1. To Federal Money.

Rule.--Reduce the Shillings, pence and farthings,

of the dividend, and the quotient will be the answer in dollars, cents and mills.

1. Reduce £349 19s. 1d. to dollars.

.9 = $\frac{1}{2}$ the shillings.

.05 = odd shilling.

4 = qrs. in 1d.

.954 = decimal.

3)349.954 D. c. m.

1166,513 = 1166, 51 3

Answer.

2. Reduce 19s. 1 $\frac{1}{2}$ d. to dollars.

.9

.05

7

.957 = decimal.

3)987 D. c.

329 = 3 29

3. Reduce 1s. to cents.

1s. = .05 then

3)05 c. m.

0,166 $\frac{2}{3}$ = 16 6 $\frac{2}{3}$

4. Reduce 1d.

1d. = 4qrs.

3)104 c. m.

001,34 = 1 34

5. Reduce 1qr.

1qr = .001041 and

3)001041

0,00,347 = 3 $\frac{1}{2}$ mi.

6. To New York and North Carolina currency.

Rule.—Add one third to the given sum.

Reduce £100 New-hampshire, &c. to New-york, &c.

£.

3)100

+ 33 6 8

£133 6 8 Ans.

7. To Pennsylvania, Newjersey, Delaware and Maryland currency.

Rule.—Add one fourth to the given sum.

Reduce £100 New-hampshire, &c. to Pennsylvania, &c.

4)100

+ 25

£125 Ans.

8. To South Carolina and Georgia currency.

Rule.—Multiply the given sum by 7, and divide the product by 9.

Reduce £100 New-hampshire, &c. to South Carolina, &c.

100

7

9)700

£77 15 6 $\frac{2}{3}$ Ans.

9. To English Money.

Rule.—Deduct one fourth from the given sum.

Reduce

119

£75 Ann/.

100

8

1200

+ 100

$$5 = 4 \times 4 = 300$$

4)825

81 5 Anf.

Reduce $\pounds 100$ New-
hampshire, &c, to Cana-
da, &c.

100

5

6)500

£83 6 8 Ans.

8. To Livres Tournois.

Reduce £100 New-
hampshire, &c. to Livres
Tournois.

100 Or, 100

 $17\frac{1}{2}$

20

700

2000

100

2

50

8) 14000

Ans. 1750

Liv. Anf. 1750 Liv.

1d. = 1 sou. 5 den. 1s. =

$$17\frac{1}{2} \text{ sous. } f = 17\frac{1}{2} \text{ livres.}$$

9. To Spanish milled Dollars.

Rule 1.—When the sum consists of pounds only annex a cypher to the pounds, and divide the whole by 20; the quotient is dollars.

Reduce

Reduce £100 New-
hampshire, &c. to Dolls.
3)1000

Dol. 333 *Ans.*

Rule 2.—When the sum consists of pounds and shillings :—Divide the pounds by 3, and the shillings by 6, not separating them in the quotient, and the quotient will be dollars.

Reduce £152 15s. 6d.
to dollars.

s. by 3 & } 152. 15/6
s. by 6. } 509 dol. and
(1/6 Ans.

Note. This article may be applied to the federal dollar, it being of the same value with a Spanish dollar.

II. To reduce Federal money to Newengland and Virginia currency.

Rule.—Multiply the Federal money by 3, and, if it consist of dollars only, cut off a figure, if of cents also, cut off three, and if of mills, 4 figures at the right hand ; then reduce the figures so cut off to farthings, each time cutting off as at first, and the left hand figures are pounds, shillings, &c.

1. Reduce 1166dolls.
51c. 8m. to Newengland
currency, or lawful money.

D. c. m.
1166,51 3
3

£ 849,9539

20

5,19,0780

12

936 = 1d. nearly

2. Reduce 45 dollars.

D.

45

3

£ 13,5

20

5,10,0

3. Reduce 12D. 7c. to
lawful money.

D. c.

12,07

3

£ 3,621

20

5,12,420

12

5,040

III. To

REDUCTION OF COINS.

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III. To reduce Newjersey, Pennsylvania, Delaware and Maryland currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Deduct one fifth from the given sum.

Reduce £100 Newjersey, &c. to Newhampshire, &c.

$$\begin{array}{r} 5)100 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 20 \\ \hline \end{array}$$

$$\begin{array}{r} \hline \pounds 80 \text{ Ans.} \\ \hline \end{array}$$

2. To Newyork and Northcarolina currency.

Rule.—Add one fifteenth to the given sum.

Reduce £100 Newjersey, &c. to Newyork, &c.

$$15 = 3 \times 5)100$$

$$\begin{array}{r} 3)20 \\ \hline \end{array}$$

$$\begin{array}{r} +6 \ 13 \ 4 + \text{giv.} \\ \hline \end{array}$$

(sum.

$$\begin{array}{r} \pounds 106 \ 13 \ 4 \text{ Ans.} \\ \hline \end{array}$$

3. To Southcarolina and Georgia currency

Rule.—Multiply the given sum by 28, and divide the product by 45.

Reduce £100 Newjersey, &c. to Southcarolina, &c.

L

$$\begin{array}{r} 100 \\ \hline \end{array}$$

$$4 \times 7 = 28$$

$$\begin{array}{r} 400 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 7 \\ \hline \end{array}$$

$$45 = 5 \times 9)2800$$

$$\begin{array}{r} 5)311 \ 2 \ 2\frac{2}{3} \\ \hline \end{array}$$

$$\text{Ans. } \pounds 62 \ 4 \ 5\frac{1}{3}$$

4. To English Money.

Rule.—Multiply the given sum by 3, and divide the product by 5.

Reduce £100 Newjersey, &c. to English money.

$$\begin{array}{r} 100 \\ \hline \end{array}$$

$$\begin{array}{r} \hline 3 \\ \hline \end{array}$$

$$\begin{array}{r} 5)300 \\ \hline \end{array}$$

$$\begin{array}{r} \hline \pounds 60 \\ \hline \end{array}$$

5. To Irish Money.

Rule.—Multiply the Newjersey, &c. by 13, and divide the product by 20.

Reduce £100 Newjersey, &c. to Irish.

$$\begin{array}{r} 100 \\ \hline \end{array}$$

REDUCTION OF COINS.

$$\begin{array}{r}
 100 \\
 \text{---} \\
 4 \times 3 + \text{the} \\
 \text{---} \text{ (giv. sum.} \\
 400 \\
 \text{---} \\
 3 \\
 \text{---} \\
 1200 \\
 + 100 \\
 \text{---} \\
 20 = 4 \times 5 \quad 1300 \\
 \text{---} \\
 4 \quad 260 \quad] \\
 \text{---} \\
 \pounds 65 \text{ Answer.} \\
 \text{---}
 \end{array}$$

5. To Canada and Nova Scotia currency.

Rule.—Deduct one third from the Newjersey, &c.

Reduce $\pounds 100$ Newjersey, &c. to Canada, &c.

$$\begin{array}{r}
 3 \quad 100 \\
 \text{---} 33 \quad 6 \quad 8 \\
 \text{---} \\
 \pounds 66 \quad 13 \quad 4 \text{ Answer.} \\
 \text{---}
 \end{array}$$

7. To Livres Tournois.

Rule.—Multiply the given sum by 14, and the product will be Livres Tournois—or multiply the sum in shillings by 7; divide the product by 10,

and the quotient will be livres, sous, &c.

Reduce $\pounds 100$ Newjersey, &c. to Livres Tournois.

$$\begin{array}{r}
 100 \text{ Or } 100 \quad \left. \begin{array}{l} 1d = 1\frac{1}{2} \text{ sous.} \\ 1s = 12 \text{ sous.} \\ 1\text{liv.} = 20 \text{ sous.} \end{array} \right\} \\
 14 \quad 20 \\
 \text{---} \quad \text{---} \\
 400 \quad 2000 \\
 100 \quad 7 \\
 \text{---} \quad \text{---} \\
 1400 \quad 10 \quad 14000 \\
 \text{Liv.} \quad | \quad \text{---} \\
 \text{Ans.} \quad | \quad 1400 \text{ as before.} \\
 \text{---}
 \end{array}$$

8. To Spanish milled Dollars.

Rule.—Multiply the given sum by $2\frac{1}{2}$.—Or multiply them by 8: Divide the product by 3. If there be shillings in the given sum, for every 7s. 6d. add 1 dollar to the quotient.

Reduce $\pounds 100$ 10s. Newjersey, &c. to Dollars.

$$\begin{array}{r}
 100 \\
 \text{---} \\
 8 \\
 \text{---} \\
 800 \\
 \text{---} \\
 266\frac{2}{3} \\
 10s. = 1\frac{1}{2} \\
 \text{---} \\
 \text{Answer. } 268 \text{ Dolls.} \\
 \text{---}
 \end{array}$$

Or,

$$\begin{array}{r} \text{Or, } 100 \\ \quad 2 \\ \hline 200 \\ 100 \times \frac{2}{3} = 66\frac{2}{3} \\ 10s. = 1\frac{1}{3} \end{array}$$

268 as before.

IV. To reduce Newyork and Northcarolina currency.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Deduct one fourth from the Newyork, &c.

Reduce £100 Newyork, &c. to Newhampshire, &c.

$$\begin{array}{r} 4)100 \\ \hline 25 \end{array}$$

£75 Answer.

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Deduct one sixteenth from the Newyork, &c.

Reduce £100 Newyork, &c. to Newjersey, &c.

$$16 = 4 \times 4)100$$

$$4)25$$

$$\hline 6 \quad 5$$

£93 15 Ans.

3. To Southcarolina and Georgia currency.

Rule.—Multiply the given sum by 7, and divide the product by 12.

Reduce £100 Newyork, &c. to Southcarolina, &c. 100

$$\begin{array}{r} 7 \\ \hline 12)700 \end{array}$$

£58 6-8 Ans.

4. To English Money.

Rule.—Multiply the given sum by 9: Divide the product by 16.

Reduce £100 Newyork, &c. to English money. 100

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$16 = 4 \times 4)900$$

$$4)225$$

£56 5 Ans.

5. To Irish Money.

Rule.—Multiply the given sum by 39: Divide the product by 64.

Reduce

REDUCTION OF COINS.

Reduce £100 New-
york, &c. to Irish mon-
ey.

$$\begin{array}{r}
 100 \\
 \hline
 6 \times 6 + \text{thrice} \\
 \hline
 \text{the given} \\
 600 \quad \text{sum.} \\
 6 \\
 \hline
 3600 \\
 + 300 = 100 \times 3 \\
 \hline
 \end{array}$$

$$64 = 8 \times 8 \quad 3900$$

$$8 \overline{) 487 \ 10}$$

£60 18 9 *Ans.*

6. To Canada and Novascotia
currency.

Rule.—Multiply the
given sum by 5, and di-
vide the product by 8.

Reduce £100 New-
york, &c. to Canada, &c.

$$100$$

$$5$$

$$8 \overline{) 500}$$

£62 10 *Ans.*

7. To Livres Tournois.

Rule.—Multiply the
given sum in shillings by
21: Divide the product
by 32, and the quotient
will be livres, sous, &c.

Reduce £100 New-
york, &c. to Livres
Tournois.

$$100$$

$$20$$

$$2000$$

$$21$$

$$2000$$

$$4000$$

$$32 = 4 \times 8 \quad 42000$$

$$4 \overline{) 5250}$$

Answer, 1312½ Livr.

Note.—1d. = 1⅓ Sous.

1s. = 12½ Sous.

1£. = 20s. Liv.

8. To Spanish milled Dollars.

Rule.—If the New-
york sum be pounds on-
ly, annex a cypher to
them, then divide by 4,
and the quotient is dol-
lars: But if it be pounds
and shillings, annex
half the shillings to the
pounds, and divide as be-
fore, and the quotient is
dollars.

Reduce £100 New-
york, &c. to Dollars.

$$4 \overline{) 1000}$$

250 dollars *Ans.*

Reduce

Reduce £100 8s. to Dollars.

$$4 \overline{)1004}$$

251 dollars *Ans.*

V. To reduce South Carolina and Georgia currency.

1. To New Hampshire, Massachusetts, Rhode Island, Connecticut and Virginia currency.

Rule.—Multiply the given sum by 9, and divide the product by 7.

Reduce £100 South Carolina, &c. to New Hampshire, &c.

$$100$$

$$9$$

$$7 \overline{)900}$$

£128 11 5½ *Ans.*

2. To New Jersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the given sum by 45, and divide the product by 28.

Reduce £100 South Carolina, &c. to New Jersey, &c.

$$1$$

$$100$$

$$9 \times 5 = 45$$

$$900$$

$$5$$

$$28 = 4 \times 7 \overline{)4500}$$

$$4 \overline{)642} \quad 17 \quad 15$$

Ans. £163 14 8½

3. To New York and North Carolina currency.

Rule.—Multiply the given sum by 12, and divide the product by 7.

Reduce £100 South Carolina, &c. to New York, &c.

$$100$$

$$12$$

$$7 \overline{)1200}$$

£171 8 6½ *Ans.*

4. To English Money.

Rule.—From the given sum deduct one twenty-eighth.

Reduce £100 South Carolina, &c. to English Money,

$$28 =$$

$$28 = 4 \times 7) 100$$

$$4) 14 \quad 5 \quad 8\frac{2}{7}$$

— 3 11 5 $\frac{1}{7}$ from the
— — — — — giv. sum.

$$\text{£ } 96 \quad 8 \quad 6\frac{2}{7} \text{ Ans.}$$

5. To Canada and Nova Scotia currency.

Rule.—Multiply the given sum by 15, and divide the product by 14.

Reduce £100 South-carolina, &c. to Canada, &c.

$$100$$

$$5 \times 3$$

$$500$$

$$3$$

$$14 = 2 \times 7) 1500$$

$$2) 214 \quad 5 \quad 8\frac{2}{7}$$

$$\text{£ } 107 \quad 2 \quad 10\frac{2}{7}$$

6. To Livres Tournois.

Rule.—Multiply the given pounds by $25\frac{1}{2}$, and the product will be livres.

Reduce £100 South-carolina, &c. to Livres.

$$100$$

Note.

$$22\frac{1}{2} \text{ s.} = 12 \text{ sous.}$$

$$13 \text{ s.} = 1 \text{ livre.}$$

$$200 \text{ s.} = 22\frac{1}{2} \text{ liv.}$$

$$200$$

$$50$$

$$\text{Ans. } 2250 \text{ Livres.}$$

7. To Irish Money.

Rule.—Multiply the given sum by 117, and divide the product by 112.

Reduce £100 South-carolina, &c. to Irish.

$$100$$

$$12 \times 9 + 9$$

(times

$$1200$$

the

$$9$$

given

sum.

$$10800$$

$$+ 100 \times 9 = 900$$

$$112 = 4 \times 4 \times 7) 11700$$

$$4) 1671 \quad 8 \quad 6\frac{2}{7}$$

$$4) 417 \quad 17 \quad 1\frac{1}{2}$$

$$\text{Ans. } \text{£ } 104 \quad 9 \quad 3\frac{3}{7}$$

8. To Spanish milled Dollars.

Rule.—Multiply the given pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars, and add them.

Reduce £100 South-carolina, &c. to dollars.

$$100$$

$$10 \times 3 = 30$$

$$1000$$

$$3$$

$$7) 2000$$

$$\text{Dollars } 428\frac{2}{7} \text{ Ans.}$$

$$\text{Note, } \frac{1}{7} = 8d.$$

Vl. To

REDUCTION OF COINS.

117

VI. To reduce English Money.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—To the English sum add one third.

Reduce £100 English, to Newhampshire, &c.

$$3)100$$

$$+ 33 \ 6 \ 8$$

$$\underline{\hspace{1cm}} \\ \text{£}133 \ 6 \ 8 \text{ Ans.}$$

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the English money by 5, and divide the product by 3.

Reduce £100 English to Newjersey, &c.

$$100,$$

$$5$$

$$3)500$$

$$\underline{\hspace{1cm}} \\ \text{£}166 \ 13 \ 4 \text{ Ans.}$$

3. To Newyork and Northcarolina currency.

Rule.—Multiply the English money by 16, and divide the product by 9.

Reduce £100 English to Newyork, &c.

$$100$$

$$4 \times 4$$

$$\underline{\hspace{1cm}} \\ 400$$

$$4$$

$$9)600$$

$$\underline{\hspace{1cm}} \\ \text{£}177 \ 15 \ 6 \frac{2}{3} \text{ Ans.}$$

4. To Southcarolina and Georgia currency.

Rule.—To the English money add one twenty-seventh.

Reduce £100 English to Southcarolina, &c.

$$27 = 3 \times 9)100$$

$$3)11 \ 2 \ 2 \frac{2}{3}$$

$$+ 3 \ 14 \ 0 \frac{2}{3}$$

$$\underline{\hspace{1cm}} \\ \text{Ans. £}103 \ 14 \ 0 \frac{2}{3}$$

5. To Irish Money.

Rule.—To the English sum add one twelfth.

Reduce £100 English money to Irish money.

$$12)100$$

$$+ 8 \ 6 \ 8$$

$$\underline{\hspace{1cm}} \\ \text{£}108 \ 6 \ 8 \text{ Ans.}$$

6. To Canada and Novascotia currency.

Rule.—To the English sum add one ninth.

Reduce

Reduce £100 English
to Canada, &c.

$$\begin{array}{r} 9)100 \\ +11 \ 2 \ 2\frac{2}{3} \\ \hline \end{array}$$

$$£111 \ 2 \ 2\frac{2}{3} \text{ Ans.}$$

7. To Livres Tournois.

Rule.—Multiply the English pounds by $23\frac{1}{3}$, and the product will be Livres.

Reduce £100 English
to Livres Tournois.

$$100 \quad \text{Note,}$$

$$23\frac{1}{3} \ 1d. = 11\frac{1}{3} \text{ sous.}$$

$$15. = 1\frac{1}{2} \text{ liv.}$$

$$300 \ 1\frac{1}{2} = 23\frac{1}{3} \text{ liv.}$$

$$200$$

$$33\frac{1}{3}$$

Liv. sou. de.

$$A. 2333\frac{1}{3} \text{ liv.} = 2333 \ 6 \ 8$$

VII. To reduce Irish Money.

1. To New Hampshire, Massachusetts, Rhode Island, Connecticut, & Virginia currency.

Rule.—Multiply the Irish sum by 16, and divide the product by 18.

Reduce £100 Irish to
New Hampshire, &c.

$$100$$

$$4 \times 4$$

$$400$$

$$4$$

$$18)1600$$

$$£122 \ 1 \ 6\frac{2}{3} \text{ Ans.}$$

2. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the Irish sum by 20, and divide the product by 18.

Reduce £100 Irish to
Newjersey, &c.

$$100$$

$$4 \times 5 = 20$$

$$400$$

$$5$$

$$18)2000 \ 153 \ 16 \ 11\frac{1}{3} \text{ Ans.}$$

$$13$$

$$11$$

$$12$$

$$20$$

$$12$$

$$70$$

$$65$$

$$13$$

$$16$$

$$13$$

$$14$$

$$11$$

$$50$$

$$13$$

$$13$$

$$39$$

$$90$$

$$14$$

$$78$$

$$13$$

$$13$$

$$11$$

$$12$$

$$1$$

3. To Livres Tournois.

Rule.—Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quotient will be sous, which divided by 20, will be livres.

Reduce £100 Irish to
Livres Tournois.

$$100 \times 20 \times 12 = 24000d.$$

$$70$$

$$2,0$$

$$39)168000(4307.6$$

$$\text{sou.}$$

$$\text{Ans. Livres, } 2153, 16\frac{2}{3}$$

$$1d = 1\frac{1}{3}\frac{1}{3} \text{ sous, } 1s = 12\frac{1}{3} \text{ sous}$$

$$1\frac{1}{2} = 21 \text{ livres, } 10\frac{1}{3} \text{ sous.}$$

4. To

REDUCTION of COINS.

129

4. To Canada and Novoscotia currency.

Rule.—To the Irish sum add one thirtyninth.

Reduce £100 Irish to Canada, &c.

39)100(2

78

22

20

39)440(11

39

50

39

11

12

39)132(3

117

15

5

— = —

39

13

5. To Southcarolina and Georgia currency.

Rule.—Multiply the Irish sum by 112, and divide the product by 117.

Reduce £100 Irish to Southcarolina, &c.

100

7 × 4 × 4 = 112

700

4

2800

4

117)11200(95 14 6 ⁴³/₁₁₇ Ans.

1053

670

585

85

6. To Newyork and Northcarolina currency.

Rule.—Multiply the Irish sum by 64, and divide the product by 39.

Reduce £100 Irish to Newyork, &c.

100

8 × 8 = 64

800

8

39)6400(164 2 ²/₃₉ Ans.

39

250

234

160

156

4

7. To

7. To English Money.

5)100

+20

£120 Ans.

Rule.—From the Irish sum deduct one thirteenth.

Reduce £100 Irish to English money.

13)100(7

91

9

20

13)180(13

13

50

39

11

12

£. s. d.

100 0 0

—7 13 10²/₁₃£92 6 1¹/₁₃*Ans.*

13)132(10

13

2

VIII. To reduce Canada and Nova Scotia currency.

1. To New Hampshire, Massachusetts, Rhode Island, Connecticut and Virginia currency.

Rule.—To the given sum add one fifth.

Reduce £100 Canada, &c. to New Hampshire, &c.

2. To New York and North Carolina currency.

Rule.—Multiply the given sum by 8, and divide the product by 5.

Reduce £100 Canada, &c. to New York, &c.

100

8

5)800

£160 Ans.

3. To New Jersey, Pennsylvania, Delaware and Maryland currency.

Rule.—To the given sum add one half.

Reduce £100 Canada, &c. to New Jersey, &c.

2)100

+50

£150 Ans.

4. To South Carolina and Georgia currency.

Rule.—From the given sum deduct one fifteenth.

Reduce £100 Canada, &c. to South Carolina, &c.

REDUCTION OF COINS.

131

$$\begin{array}{r}
 25 = 3 \times 5 \ 100 \\
 \hline
 3 \ 20 \\
 \hline
 -6 \ 13 \ 4 \\
 \hline
 \pounds 93 \ 6 \ 8 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 100 \\
 \hline
 7 \times 3 = 21 \\
 \hline
 700 \quad 1d. = 12 \text{ Sous.} \\
 3 \quad 1s. = 20 \text{ Sous.} \\
 \hline
 2100 \text{ Ans.}
 \end{array}$$

5. To English Money.

Rule.—From the given sum deduct one tenth.

Reduce $\pounds 100$ Canada, &c. to English money.

$$\begin{array}{r}
 10 \ 100 \\
 \hline
 -10 \\
 \hline
 \pounds 90 \text{ Ans.}
 \end{array}$$

6. To Irish Money.

Rule.—From the given sum deduct one fortieth.

Reduce $\pounds 100$ Canada, &c. to Irish money.

$$\begin{array}{r}
 40 \ 100 \\
 \hline
 -2 \ 10 \\
 \hline
 \pounds 97 \ 10
 \end{array}$$

7. To Livres Tournois.

Rule.—Multiply the given pounds by 21, and the product will be Livres.

Reduce $\pounds 100$ Canada, &c. to Livres Tournois.

8. To Federal, or Spanish Dollars.

Rule.—Reduce the given sum to shillings: Divide them by 5. Or, Multiply the pounds by 4, and if there be shillings, turn them into Dollars, and add them to the product.

Reduce $\pounds 100$ Canada, &c. to Dollars.

$$\begin{array}{r}
 100 \quad 155 \ 15 \\
 20 \quad 4 \\
 \hline
 5 \ 2000 \quad 620 \\
 \hline
 +3 = 15s. \\
 D. 400 \text{ Ans.} \\
 \text{Dol. 623 Ans.}
 \end{array}$$

IX. To reduce Livres Tournois.

1. To Newhampshire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Multiply the Livres by 3; divide the product by 35, and the quotient will be pounds: Or, Multiply the Livres by

by 8: Divide the product by 7, and the quotient will be shillings.

Reduce 1750 Livres to Newhampshire, &c. currency.

$$\begin{array}{r}
 1750 \quad \text{Or, } 1750 \\
 \underline{2} \quad \quad \quad 8 \\
 \text{---} \quad \text{£.} \quad \text{---} \\
 35) 3500 (100 \text{ Ans. } 7) 14000 \\
 \underline{35} \quad \quad \quad \underline{14} \\
 00 \quad \quad \quad 2,0) 200,0 \\
 \quad \quad \quad \underline{\quad} \\
 \quad \quad \quad \text{£100}
 \end{array}$$

(as before.

1. To Newyork and Northcarolina currency.

Rule.—Multiply the Livres by 32: Divide the product by 21, and the quotient will be shillings.

Reduce 1312½ Livres to Newyork, &c. currency.

$$\begin{array}{r}
 1312,5 \\
 \underline{32} \\
 \text{---} \\
 26250 \\
 39375 \\
 \underline{\quad} \quad 2,0 \\
 21) 42000 (200,0 \\
 \quad \quad \quad \text{£100 Ans.}
 \end{array}$$

3. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Divide the Livres by 14, and the quotient will be pounds: Or, Multiply the Livres by

10: Divide the product by 7, and the quotient will be shillings.

Reduce 1400 Livres to Newjersey, &c. currency.

$$\begin{array}{r}
 1400 \\
 \underline{10} \\
 \text{---} \quad \text{Or, } \text{£.} \\
 7) 14000 \quad 14) 1400 (100 \\
 \underline{14} \\
 2,0) 200,0 \quad \underline{14} \\
 \quad \quad \quad \text{---} \\
 \quad \quad \quad \text{£100 Ans.}
 \end{array}$$

4. To Southcarolina and Georgia currency.

Rule.—Multiply the Livres by 2, divide the product by 45, and the quotient will be pounds. Or, deduct one ninth, and the remainder will be shillings.

Reduce 2250 Livres to Southcarolina, &c. currency.

$$\begin{array}{r}
 2250 \quad \text{Or,} \\
 \underline{2} \quad \quad \quad 9) 2250 \\
 \text{---} \quad \quad \quad \underline{250} \\
 \text{£.} \quad \quad \quad \text{---} \\
 45) 4500 (100 \text{ Ans.} \\
 \underline{45} \quad \quad \quad 2,0) 200,0 \\
 \quad \quad \quad \underline{\quad} \\
 \quad \quad \quad 00 \quad \quad \quad \text{£100} \\
 \quad \quad \quad \text{(as before,}
 \end{array}$$

5. To English Money.

Rule.—Deduct one seventh from the Livres, and

and the remainder will be shillings.

Reduce $2333\frac{1}{3}$ Livres to English Money.

$$\begin{array}{r} 7) 2333\frac{1}{3} \\ - 333\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 2) 2000 \\ \hline \end{array}$$

£ 100 Ans.

6. To Irish Money.

Rule.—Reduce the livres to sous; then multiply them by 39. Divide this product by 70, and the quotient will be pence.

Reduce 2153 liv. $16\frac{1}{3}$ sous, to Irish Money.

liv. sous.

$$\begin{array}{r} 2153 \quad 16\frac{1}{3} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 43 \quad 76\frac{1}{3} \\ 39 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \quad 7720 \\ 129228 \\ \hline \end{array}$$

$$\begin{array}{r} 7,0) 68000,0 \\ \hline \end{array}$$

$$\begin{array}{r} 12) 24000 \\ \hline \end{array}$$

$$\begin{array}{r} (2,0) 200,0 \\ \hline \end{array}$$

£ 100 Ans.

7. To Federal or Spanish Dollars.

Rule.—Multiply the

M

livres by 4; divide the product by 21, and the quotient will be Spanish or Federal Dollars.

Reduce 1000 Livres to Dollars.

$$\begin{array}{r} 1000 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 21) 4000 \\ 21 \\ \hline \end{array}$$

(190 Fed. or (Span. Dol.

$$\begin{array}{r} 190 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 189 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 21 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 21 \\ \hline \end{array}$$

X. To reduce Federal or Spanish Dollars.

1. To Newhamphire, Massachusetts, Rhodeisland, Connecticut and Virginia currency.

Rule.—Multiply the Dollars by 3, and double the right hand figure of the product, for shillings; the left hand figures are pounds.

Reduce 529 Dollars to Newhamphire, &c.

$$\begin{array}{r} 529 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 1587 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 158 \quad 14 \text{ Ans.} \\ \hline \end{array}$$

s. To

2. To Newyork and North-carolina currency.

Rule.—Multiply the Dollars by 4; double the right hand figure of the product for shillings, and the left hand figures are pounds.

Reduce 529 Dollars to Newyork, &c.

$$\begin{array}{r} 529 \\ 4 \end{array}$$

£ 211 12 *Ans.*

3. To Newjersey, Pennsylvania, Delaware and Maryland currency.

Rule.—Multiply the Dollars by 3, and divide by 8.

Reduce 529 Dollars to Newjersey, &c.

$$\begin{array}{r} 529 \\ 3 \end{array}$$

£ 158 7 6 *Ans.*

$$\begin{array}{r} 78 \\ 8 \end{array}$$

$$\begin{array}{r} 72 \\ 8 \end{array}$$

$$\begin{array}{r} 67 \\ 8 \end{array}$$

$$\begin{array}{r} 64 \\ 8 \end{array}$$

$$\begin{array}{r} 3 \end{array}$$

4. To Southcarolina and Georgia currency.

Rule.—Multiply the Dollars by 7, and divide by 30.

Reduce 529 Dollars to Southcarolina, &c.

$$\begin{array}{r} 529 \\ 7 \end{array}$$

3,0)370,3

£ 123 12 *Ans.*

- *5. To English Money, at 4s. 6d. per Dollar.

Rule.—Multiply the Dollars by 9, and divide by 40.

Reduce 529 Dollars to English money.

$$\begin{array}{r} 529 \\ 9 \end{array}$$

4,0)476,1

£ 119 12 *Ans.*

6. To Canada and Novascotia currency.

Rule.—Divide the Dollars by 4.

Reduce 529 Dollars, to Canada, &c.

$$\begin{array}{r} 4)529 \end{array}$$

£ 132 12 *Ans.*

7. To Livres-Tournois.

Rule.—Multiply the Dollars

* *Note.* That in England Dollars are Bullion, that is, they are bought and sold by weight, and their value varies as other articles of merchandise.

REDUCTION OF COINS.

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Dollars by $5\frac{1}{4}$, and the product will be Livres :

Or, Multiply them by 21, divide by 4, and the quotient will be Livres.

Reduce 100 Spanish Dollars to Livres.

$$\begin{array}{r} 100. \quad \text{Or,} \\ 5\frac{1}{4} \quad 100 \\ \hline 21 \\ 500 \\ 100 \times \frac{1}{4} = 25 \quad 4) 2100 \\ \hline \end{array}$$

Ans. 525 liv. 525 as (before

Note, $\left\{ \begin{array}{l} 1 \text{ Cent} = 1\frac{1}{20} \text{ Sous.} \\ 1 \text{ Dime} = 10\frac{1}{2} \text{ Sous.} \\ 1 \text{ Dollar} = 5\frac{1}{4} \text{ Livres.} \end{array} \right\}$

The pound	{ Sterling Newhampshire, &c. Newyork, &c. Newjersey, &c. Southcarolina, &c.	{ contains	1718 $\frac{2}{3}$	{ Grains of fine Silver.
			1289	
			966 $\frac{1}{4}$	
			1031 $\frac{1}{4}$	
			1657,366	

{ Ditto, 1858 Ditto, 1893 $\frac{1}{2}$ Ditto, 1045 Ditto, 1314 $\frac{2}{3}$ Ditto, 1791,819	{ Grains of Stand. Silver.	{	The proportion of alloy being $\frac{3}{7}$ of the fine Silver.

The { Dollar } contains { 375,64 } Grains
Federal { Eagle } { 246,268 } of fine
{ Silver } 409,78 grs. of Stand. Silver.
{ Gold } 268,659 grs. of Stand. Gold.

The alloy being $\frac{1}{11}$ of the fine { Silver.
Gold.

The Subdivisions are in the same proportion.

RECAPITULATION

RECAPITULATION of the

*Seek the given Coin in the left hand column, then cast
required Coin, and you will have*

		1	2	3	4	5
		Newh. Mas. R. I. Connect. Virginia.	Newjer. Pennsyl. Delaw. Maryl.	Newyor. Northca.	Southca. Georgia.	Sterling.
1	Newh. Mas. R. Connec. & Virg		Add $3\frac{1}{2}$ Page 118.	Add $\frac{1}{2}$ Page 118.	Mult. by 7, divide Prod. by 9. P. 118.	Deduct $\frac{1}{2}$ Page 118.
2	Newjer. Pennsyl. Delaw. Maryl.	Deduct $\frac{1}{2}$ Page 121.		Add $\frac{1}{3}$ Page 121.	Mult. by 28, div. Prod. by 45. P. 121.	Mult. by 3, divide Prod. by 5. P. 121.
3	Newy. & Northc.	Deduct $\frac{1}{4}$ Page 123.	Deduct $\frac{1}{16}$ Page 123.		Mult. by 7, & div. Prod. by 12. P. 123.	Mult. by 9, & div. Prod. by 16. P. 123.
4	Southc. & Georgia	Mult. by 9 & div. Prod. by 7. Page 125.	Mult. by 45, & div. prod. by 28. P. 125.	Mult. by 12, & div. by 7. Page 125.		Deduct $\frac{1}{16}$ Page 125.
5	Sterl.	Add $\frac{1}{3}$ Page 127.	Mult. by 5, & div. Prod. by 3. P. 127.	Mult. by 16, & div. Prod. by 9. P. 127.	Add $\frac{1}{27}$ Page 127.	
6	Irish.	Mult. by 16, & div. Prod. by 13. P. 128.	Mult. by 20, & div. by 13. Page 128.	Mult. by 64, & div. by 39. Page 129.	Mult. by 112, & div. by 117. Page 129.	Deduct $\frac{1}{17}$ Page 130.
7	Canada & Novasc.	Add $\frac{1}{5}$ Page 130.	Add $\frac{1}{2}$ Page 130.	Mult. by 8, & div. prod. by 5. Page 130.	Deduct $\frac{1}{15}$ Page 129.	Deduct $\frac{1}{16}$ Page 131.
8	Livres Tourn.	Mult. by 2 div. by 35. or by 8, & d. 7 & q. 153.	Divide by 14. Page 132.	M. by 32, div. by 21. & quo. is shillings.	Deduct $\frac{1}{3}$ & rem. is shills. Page 132.	Deduct $\frac{1}{5}$ & rem. is shills. Page 132.
9	Federal or Spanish Dollars.	M. 3, dou. rt. hd. pro for s. left are pounds.	Mult. by 3, & divide Prod. by 8. Page 134.	M. 4, dou. rt. hd. pro. f. for s. left are pounds.	Mult. by 7, & div. Prod. by 30. P. 134.	Mult. by 9, & div. by 40. Page 134.

preceding RULES of EXCHANGE.

your eye to the right hand, till you come under the the Rule, with the Reference.

6	7	8	9
Irish.	Canada. Novaasco.	Livres Tournois.	Federal or Spanish Dollars.
Multip. by 13, div. by 16. Page 119.	Multip. by 5, div. by 5. Page 119.	Multiply by $17\frac{1}{2}$. Page 119.	If £'s. only annex a 0, & di. by 3. If £'s. & s. an half s. & div. as bef. q. is d. re. s. & c.
Multip. by 13, divide Prod. by 20. P. 121.	Deduct $\frac{1}{3}$. Page 122.	Multiply by 14. Page 122.	Multiply by $2\frac{2}{3}$. Page 122.
Multip. by 39, & div. Prod. by 64. Page 123.	Multip. by 5, & divide by 8. Page 124.	Multiply the sum in shillings by 21, divide Product by 32. Page 124.	If £'s. only annex a 0, & di. by 4. If £'s. & s. an. half the s. & div. as before. P. 124.
Multip. by 117, & div. Prod. by 112. P. 126.	Multip. by 15, & div. by 14. Page 126.	Multiply pounds by $22\frac{1}{2}$. Page 126.	Mult. pounds by 30, div. prod. by 7, & if any s. turn them to dols. & add them.
Add $\frac{1}{2}$. Page 127.	Add $\frac{1}{5}$. Page 127.	Multiply pounds by $23\frac{1}{2}$. Page 128.	Mult. by 40, divide Prod. by 9, & the rem. is six pences.
	Add $\frac{1}{39}$. Page 129.	Multiply the sum in pence by 70, div. by 39, & quotient is s. which div. by 20.	M. by 160, di pro. by 39, q. is dol. & unit in rem. is $1\frac{1}{2}$ d. Irish, or 2 cents nearly.
Deduct $\frac{1}{30}$. Page 131.		Multiply by 21. Page 131.	Multiply by 4. Page 131.
Re. them to shil. M. by 39, div. by 70, & q. is d.	Deduct $\frac{1}{21}$ & the remaind. is shillings.		Multiply by 4, and divide Product by 21. Page 133.
Div. by 4 & ded. $\frac{1}{30}$ of the quotient.	Div. by 4. Page 134.	Multiply by 54. Page 134.	

The Standard Weight of an EAGLE is *11 pwt.*
 $4\frac{2}{3}$ *grs.*—Half ditto, 5 *pwt.* $14\frac{1}{3}$ *grs.*—A DOLLAR
 17 *pwt.* $12\frac{3}{4}$ *grs.*—Half ditto, 8 *pwt.* $12\frac{1}{4}$ *grs.*—A
 DOUBLE DIME 3 *pwt.* $9\frac{3}{4}$ *grs.*—A DIME 1 *pwt.*
 $16\frac{2}{3}$ *grs.*

TABLE of the value of several Pieces of Coin, in the
 FEDERAL COIN, and the several Currencies of the
 UNITED STATES.

	Federal Coin.	N. Hamp. shire, Mas- sachusetts R. Island, Connecti- cicut, Vir- ginia.	New York and North Carolina	New Jersey, Pennsylvania, Delaware and Maryland.	South Carolina and Georgia.
	Cents	£. s. d.	£. s. d.	£. s. d.	£. s. d.
$\frac{1}{10}$ of a Doll.	0,06 $\frac{1}{2}$	4 $\frac{1}{2}$	6	5 $\frac{1}{2}$	3 $\frac{1}{2}$
$\frac{1}{20}$ of a Pistareen	0,10	7 $\frac{1}{2}$	9 $\frac{1}{2}$	9	5 $\frac{1}{2}$
		Vir. 8			
$\frac{1}{10}$ of a Dollar	0,11 $\frac{1}{2}$	8	10 $\frac{1}{2}$	10	6 $\frac{1}{2}$
$\frac{1}{20}$ of ditto	0,12 $\frac{1}{2}$	9	11 $\frac{1}{2}$	11 $\frac{1}{2}$	7
A Pistareen	0,20	12 $\frac{1}{2}$	14 $\frac{1}{2}$	16	11 $\frac{1}{2}$
		Vir. 14			
Eng. Shill.	0,22 $\frac{1}{2}$	14	17 $\frac{1}{2}$	18	10 $\frac{1}{2}$
$\frac{1}{10}$ of a Doll.	0,25	16	20	21 $\frac{1}{2}$	12
Half ditto.	0,50	30	40	43	24
A Dollar	1,00	60	80	86	48
Eng. Crown.	1,11 $\frac{1}{2}$	68	N.Y. 90 N.C. 89	94	52 $\frac{1}{2}$
<i>pwt. gr.</i>					
Fr. G. 5 s	4,62 $\frac{2}{3}$	176	116 0	114 6	115
In Mass. 3 6	4,55 $\frac{1}{3}$	174			
En. Gu. 5 6	4,66 $\frac{2}{3}$	180	117 0	115 0	119
In S. C. 3 7					110
$\frac{1}{2}$ Joha. 9 0	8,00	280	34 0	30 0	174
Pistole 4 5					
In Mass. 4 3	3,66 $\frac{2}{3}$	120	18 0	17 0	176
Moid. 6 18	6,00	116 0	26 0	25 0	80
Doub. 17 0	14,66 $\frac{2}{3}$	480	516 0	512 0	310 0

DUODECIMALS.

139

CROSS MULTIPLICATION ;

Or,

DUODECIMALS,

Is a Rule made use of by Workmen and Artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus ; inches or primes ($'$), seconds ($''$), thirds ($'''$), fourths ($''''$), &c.

This Method of multiplying is not confined to *twelves* ; but may be greatly extended : For any number, whether its inferior denominations decrease from the integer in the same ratio, or not, may be multiplied crosswise.

RULE 1.—Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each, under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superior, and for other numbers accordingly.

3. In the same manner multiply all the multiplicand by the primes or second denomination in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will be the answer required.

EXAMPLES.

EXAMPLES.

1. Multiply
- $2\frac{1}{2}$
- feet by
- $2\frac{1}{2}$
- feet.

F. ' "

2 6

2 6

5 0

1 3 0

6 3

Ans.

Or, thus.

 $2\frac{1}{2}$ $2\frac{1}{2}$

5

1 3

1 3

Or, thus.

2, 5

2, 5

125

50

Ans. 6, 25 Square Feet.

Ans. $6\frac{1}{2}$ square feet = 6 feet, 36 inches.So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{2}$ of a square foot.

2. Multiply 9f. 8' 6" by 7f. 9' 3".

f. ' "

9 8 6

7 9 3

67 11 6

7 3 4 6"

2 5 1 6"

75 5 3 7 6

Ans.

= Product by the feet in the mul.

= ditto by the primes. (multiplier.

= ditto by the seconds.

3. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide?

Ans. 24f. 10' 11"

4. How many cubic feet in a stick of timber 13 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick?

Ans. 35f. 6' 8" 6"

The two following questions are Sexagesimals.

5. If two places differ in longitude
- $2^{\circ} 12'$
- ; what is their difference of time?

Mult. $2^{\circ} 12' 00'' 00'''$

by

 $3^{\circ} 59' 20'''$

the time, in which the Sun (passes through 1 degree.

 $8^{\circ} 46' 32'''$

Ans.

6. Two

6. Two places differ in longitude $31^{\circ} 27' 30''$; what is the difference, in time, of the Sun's coming to the meridian of those places, the Sun passing through 15° in an hour?

$31^{\circ} 27' 30''$

$4' 00''$

In 4' of a solar day, or day of 24 hours,
(the Sun passes 1 degree.

$2 \quad 6' 30'' 00''$ Ans.

SINGLE RULE OF THREE DIRECT.

The Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three Direct.

But if *more* require *less*, or *less* require *more*, it belongs to the Rule of Three Inverse.*

RULE† 1.—State the question by making that number, which asks the question, the third term, or putting it in the third place: That, which is of the same name or quality as the demand, the first term;

* *More* requiring *more*, is, when the third term is greater than the first, and requires the fourth term to be greater than the second. And *less* requiring *less*, is, when the third term is less than the first, and requires the fourth term to be less than the second.

Also, *more* requiring *less*, is, when the third term is greater than the first, and requires the fourth term to be less than the second. And *less* requiring *more*, is, when the third term is less than the first, and requires the fourth term to be greater than the second.

† This Rule, on account of its great and extensive usefulness, is sometimes called the *Golden rule of Proportion*; for, on a proper application of it and the preceding rules, the whole business of Arithmetic, as well as every mathematical inquiry, depends. The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: Thus, the quantity of goods, bought, is in proportion to the money laid out;—the space, gone over by an uniform motion, is in proportion to the time, &c.

term ; and that, which is the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together ; divide the product by the first, and the quotient will be the answer to the question, which (as also the remainder) will be in the same denomination you left the second term in, and which may be brought into any other denomination required.

Two or more statings, are sometimes necessary, which may always be known from the nature of the question.

The method of proof is by inverting the question.

1. If

It has been shewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole ; and in Division, that the price of the whole, divided by the quantity, is the price of one : Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain, that the answer, found by this rule, will be the same as that, found by Multiplication of Money ; and where one is the last term of the proportion, it will be the same as that, found by Division of Money.

In like manner, if the first term be any number whatever, it is plain that the product of the second and third terms will be greater than the true answer required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds an unit ; consequently, this product, divided by the first term, will give the true answer required.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided, and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first ; multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first ; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

SINGLE RULE OF THREE.

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1. If 6lb of sugar cost 9s. what will 30lb cost at the same rate?

Here, the first clause (if 6lb of sugar cost 9s.) supposes the rate; then follows the question; What will 30lb cost?

30lb which moves the question, is the 3d term. — 6lb, the same kind, is the 1st — and 9 shillings the 2d.

As 6 : 9 :: 30 : the *Ans*.

9

6)270

Ans. 45s. = £2 5s.

Again — By inverting the order of the question, it will be,

2. If 9s. buy 6lb of sugar, how much will £2 5s. buy at that rate?

s. lb s.

As 9 : 6 :: 45 the *Ans*.

6

9)270

30lb, *Ans*.

Again 3. If 30lb of sugar be worth £2 5s. how much may I buy for 9s.?

As

Note, The term which asks or moves the question, has generally some words like these before it, viz. What will? What cost? How many? How far? How long? or, How much? &c.

OBSERVATIONS.

1. The fourth number is always found in the same name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest, which it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder after the division, and the quotient be not the least denomination of its kind; then, multiply the remainder by that number, which one of the same denomination with the quotient

As 45 : 30 :: 9 : the *Ans.*

9

45)270(6 lb. *Ans.*

Again 4. Suppose £2 5s. will buy 30lb. of sugar : What will 6lb. of the same sugar cost ?

lb. s. lb.

As 30 : 45 :: 6 : the *Ans.*

6

3|0)27|0

9s. *Ans.*

N. B. The three last questions are only the first varied, being purely to shew how any question in this Rule, may be inverted.

5. If 5yds. of cloth cost £1 10s. what will 20yds. ditto come to ?

yds. s. yds.

As 5 : 30 :: 20

30 ÷ 5 = 6

£1 10s.

20

5)30s.

6 Quo.

2,0)12,0s. = £6 *Ans.*

Here, I divide the 2d term by the 1st, and mul-

quotient contains of the next less, and divide this product again by the first number ; and thus proceed, till the least denomination be found, or till nothing remain.

5. If the first number be greater than the product of the second and third ; then bring the second to a lower denomination.

6. What

multiply the quotient into the 3d, for the answer.

Again 6. As 5 : 30 :: 20

20 ÷ 5 = 4

120s. = £6

Here, I divide the 3d term by the 1st, and multiply the quotient into the 2d, for an answer.

7. If 20yds. cost £120, how many yards may I have for £80 ?

£. yds. £.

As 120 : 20 :: 80

120 ÷ 20 = 6 Quotient &

80 ÷ 6 = 5 yds. *Ans.*

Here, I divide the 1st term by the 2d, and then the 3d term by that quotient for the answer.

Again, £. yds. £.

8. As 120 : 20 :: 80

120 ÷ 80 = 4 Quotient,

20 ÷ 4 = 5 yds. *Ans.*

Here, I divide the 1st term by the 3d, and then the 2d term by that quotient for the answer.

9. If

SINGLE RULE OF THREE.

145

9. If 1 Cwt. of tobacco cost £5 12 9½; what will 8 Cwt. ditto cost at that rate?

Cwt. £. s. d. Cwt.
As 1 : 5 12 9½ :: 8

Ans. £45 2 4

Here is no need of reducing the middle term; because it can be performed by compound multiplication, the first term being an unit.

10. If 8 Cwt. of tobacco cost £45 2s. 4d. what is that per Cwt.?

£. s. d.
8)45 2 4

Ans. £5 12 9½

Here, there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If 57yds. cost £69, what will 9yds. cost at that rate?

yds. £. yds.
As 57 : 69 :: 9

57)621(10£. Ans.
57
51
10

57)1020(17s.

57

450

399

51

12

57)512(8d.

57

110)1074(9

57)168(2

114

Here, all the terms being whole numbers, there is no need of reducing the middle one till after stating.

12. If

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the contents of one be reduced to the lowest name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it most generally happens) put the separate content of each properly under one another, then add them together, and you will have the whole quantity.

12. If 9C. 3qrs. sugar cost £27 17s. 6d. what will 2C. 1qr. 1lb. cost?

9C. 3qrs.	£27 17s. 6d.	2C. 1qr. 1lb.
4	20	4
—	—	—
39	557	9
28	13	28
—	—	—
312	6690	73
78		19
—		—
1092		263

As 1092 : 6690 :: 263 : the Answer.

263

2007

4014

1338

12

1092)1759470(1611

1092

210)1343d.

6674

6552

—

1227

1092

—

1350

1092

—

238

—

1092)1092(1

1092

—

—

—

—

—

—

—

—

—

—

—

—

—

—

£6 14 3d. Answer.

Note 1. If you look at the first

ing, you will see that the first

and third terms are of the same

kind, but of different denomina-

tions, and therefore are reduced

1350 to the same name or denomina-

tion, and, that the demand of the

question lies on the third term.

2. That the middle term, be-

ing given in pounds, shillings and

pence, is reduced to pence.

1092)1092(1

1092

—

—

—

—

—

—

—

13. If my income be 109 guineas per annum, I

desire

desire to know what I may spend per day, so that I may lay up £45 at the year's end?

(Ans. £0 5 10½ $\frac{1}{3}$ per day.

Note 1. You must subtract £45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

D. Guin. £. D.

As 365 : 109 — 45 :: 1 : the Answer.

14. If my salary be £43 12s. 5d. per annum, what does it amount to per week?

D. £. s. d. D.

As 365 : 43 12 5 :: 7 : 16s. 8½d. Ans.

15. Suppose my income to be 16s. 8½d. per week, what is it per annum? Ans. £43 12 5.

D. £. s. d. D.

As 7 : 16 8½ :: 365 : the Answer.

16. If I am to pay 1s. 7d. per week for pasturing a cow; what must I give per week for 37 cows?

C. s. d. C. £. s. d.

As 1 : 1 7 :: 37 : 2 18 7 Ans.

17. How many yards of cloth may be bought for £57 13s. whereof 97 yards cost £3 15s. 5½d.?

£. s. d. Yds. £. s. d. Yds. qr. n.

As 3 15 5½ : 97 :: 57 13 : 145 0 2 $\frac{2}{3}$ Ans.

18. If I buy 57 yards of cloth for 49 guineas; what did it cost per ell English?

Yds. Guin. Yds.

As 57 : 49 :: 1 : £1 10s. 1 $\frac{1}{3}$ d. Ans.

19. A merchant, failing in trade, owes in all £3475, and has in money and effects but £2316 13s. 4d. Now, supposing his effects are delivered up; pray, what will each creditor receive on the pound?

£. £. s. d. £.

As 3475 : 2316 13 4 :: 1 : £0 13s. 4d. Ans.

20. A owes B £3475, but B compounds with him for 13s. 4d. on the pound; pray, what must he receive for his debt?

£. £. s. d. £.

As 3475 : 13 4 :: 1 : £0 13s. 4d. Ans.

As £. s. d. £. s. d. Answer.
 As 1 : 13 4 :: 8475 : 2316 13 4

21. If the distance from Newburyport to York be 31 miles; I demand, how many times a wheel, whose circumference is $15\frac{1}{2}$ feet, will turn round in performing the journey?

Feet. Cir. M. Cir.

As $15\frac{1}{2}$: 1 :: 31 : 10560 times, Answer.

22. Bought 9 chests of tea, each weighing 3 C. 2 qrs. 21 lb, at £4 9s. per Cwt. what came they to?

Cwt. £. s. C. qr. lb. Answer.

As 1 : 4 9 :: 3 2 21 X 9 : 147 13 8

23. What will $37\frac{1}{2}$ gross of buttons, at $9\frac{1}{2}$ d. per dozen, come to?

Doz. d. Gross. £. s. d.

As 1 : $9\frac{1}{2}$:: $37\frac{1}{2}$: 17 16 3 Answer.

24. A farm, containing 125 A. 8 R. 27 P. is valued at 20 9s. per acre; what is the yearly rent of that farm?

A. £. s. A. R. P. £. s. d.

As 1 : 3 9 :: 105 3 27 : 1254 14 11 Answer.

25. If a ship cost £537, what are $\frac{1}{4}$ of her worth?

Eigh. £. s. d.

As 8 : 537 :: 3 : 201 7 6 Answer.

26. If $\frac{7}{16}$ of a ship cost £849; what is the whole worth?

Sixt. £. Sixt. £. s. d.

7 : 849 :: 16 : 1971 14 3 Answer.

27. Bought a cask of wine at 77 per gallon, for 125 dollars; how much did it contain?

Gal. Dol. Gal. qt. pt.

As 4 7 : 1 :: 125 : 163 2 1 Answer.

28. What comes the insurance of £537 15s. 10d. at $\frac{1}{4}$ per centum?

£. £. £. s. d.

As 100 : 41 :: 537 15 : 24 3 Answer.

29. What comes the commission of £785 on 31 guineas per cent?

£. Guin. £. s. d.

As 100 : 31 :: 785 : 28 9 3 Answer.

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards; each package contained 8 parcels; each parcel, 12 pieces; and each piece, 20 yards; what came the whole to, and what per yard?

Yds. Gain. Pack. £.

As 7 : 3 :: 9 : 10968 *Ans. for the whole cost.*

Yds. Guin. Yd.

As 7 : 3 :: 1 : £0 12s. *Answer per yard.*

31. A merchant bought 49 tons of wine for £273; freight cost £27—duties £12—cellar £9 10s. other charges £15, and he would gain £55 10s. by the bargain; what must I give him for 23 tons?

Tons. £. £. £. £. s. £ £ s. Tons. £.

As 49 : 273+27+12+9 10+15+55 10 :: 23 : 184 *Answer.*

32. If £100 gain £6 in a year, what will £475 gain in that time?

As £100 : £6 :: £475 : £28 10s. *Answer.*

33. The earth being 360 degrees in circumference, turns round on its Axis in 24 hours; how far does it turn in one minute, in the 45th parallel of Latitude: The degree of Longitude in this Latitude being about 51 statute miles?

H. D. M. M. Miles.

As 24 : 360 X 51 : 1 : 127 *Answer.*

34. Shipt for the Westindies 225 quintals of fish, at 15s 6d per quintal; 37000 feet of boards, at 8d dollars per 1000; 12000 shingles, at a half guinea per 1000; 19000 hoops at 1d dollar per 1000, and 53 half joes; and, in return, I have had 3000 gallons of rum, at 12s 3d per gallon; 2700 gallons of molasses, at 5s 4d per gallon; 15000 lb of coffee, at 8 1/2d per lb, and 29 Cwt. of sugar, at 12s 3d per Cwt. and my charges on the voyage were £27 12s. Pray, did I gain or lose, and how much, by the voyage?

Ans. lost £174 9s. 9d.

35. If a staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the height of that steeple, whose

whose shade, at the same time, measures 198 feet?

F. Sh. ft. hei. F. Sh. ft. hei.
 As 7 : 4 :: 198 : 1134 *Answer.*

36. * Suppose a tax of £755 be laid on a town, and the inventory of all the estates in the town amounts to £9345, what must A pay, whose estate is £149?

£. £. £. £.
 As 9345 : 755 :: 149 : 12 0 9 *Answer.*

37. If 50 gallons of water, in one hour, fall into a cistern, containing 230 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled?

Gal. Gal. H. Gal. H.

As 50—35 : 1 :: 230 : 25 $\frac{1}{2}$ *Answer.*

38. A Butcher went with £416, to buy cattle: Oxen at £22 each, cows at £4, steers at £3 10s. and calves at £2 10s. and of each a like number; how many of each could he purchase with that sum?

£. £. £. s. £. s. each. £. each.
 As 22+4+3 10+2 10 : 1 :: 416 : 19 *Answer.*

39. Said Harry to Dick, my purse and money are worth 3 $\frac{1}{2}$ guineas; but the money is worth eleven

* It may not be amiss to shew the general method of assessing town or parish taxes.

First, then, an inventory of the value of all the estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns: The most concise way is, then, to make the total value of the inventory the first term; the tax to be assessed, the second; and $\frac{1}{2}$ the third, and the quotient will shew the value on the pound: *Secondly*—Make a table, by multiplying the value on the pound by 1, 2, 3, 4, &c. *Thirdly*—From the inventory take the real and personal estates of each man, and find them separately, in the table; which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, as otherwise, before stating, to find the value on the pound, you must deduct the sum of the average tax from the whole sum to be assessed: For which average, you must have a separate column, as well as for the real and personal estates.

SINGLE RULE OF THREE.

43. If the earth revolves 366 times in 365 days, in what time does it perform one revolution?

Rev. Days. Rev.

As 366 : 365 :: 1 : 23h. 56' 3" 56" + = 1 (Siderial day.*

44. If the earth makes one complete revolution in 23h. 56' 3" +, in what time does it pass through one degree?

As 366° : 23h. 56' 3" :: 1° : 3' 59" 20" *Answer.*

45. If the earth performs its diurnal revolution in a Solar † day, or 24 hours; in what time does it move one degree?

As 360° : 24h. : 1° : 4' *Answer.*

46. Sold

TABLE.

£.	£.	s.	d.	£.	£.	s.	d.	£.	£.	s.	d.
1	is	—	0 6	20	is	0	10 —	200	is	5	—
2	—	—	1 0	30	—	0	15 —	300	—	7	10 —
3	—	—	1 6	40	—	1	— —	400	—	10	—
4	—	—	2 0	50	—	1	5 —	500	—	12	10 —
5	—	—	2 6	60	—	1	10 —	600	—	15	—
6	—	—	3 0	70	—	1	15 —	700	—	17	10 —
7	—	—	3 6	80	—	2	— —	800	—	20	—
8	—	—	4 0	90	—	2	5 —	900	—	22	10 —
9	—	—	4 6	100	—	2	10 —	1000	—	25	—
10	—	—	5 0								

Now, to find what A's rate will be,

His real estate being £376: I find
by the table, that £300 is £7 10s.
that £70 is — — — — 2 15
and that £6 is — — — — 3

Therefore, his real estate is £9 8

In like manner, I find the
tax for personal estate to be } 3 14 6d

His 3. polla, at 5s. 3d. each, are 15 9

£13 18 3

Real.	Personal.	Polla.	Total.
£. s. d.	£. s. d.	£. s. d.	£. s. d.
9 8 0	3 14 6	0 15 9	13 18 3

* A Siderial day is the space of time which happens between the departure of a Star from, and its return to the same meridian again.
† The Solar day is that space of time which intervenes between the Sun's departing from any one meridian to the same again.

46. Sold a cargo of flax seed in Ireland, for £1795 10s. Irish money; what does that amount to, in Massachusetts currency, £81 5s. Irish being equal to £100 Massachusetts?

Irish. Mass. Irish. Mass.

As £81½ : £100 :: £1795½ : £2209 16s. 11d. *Ans.*
Or, As 13 : 1795½ :: 16 : £2209 16s. 11d. as before, because £13 Irish are equal to £16 Massachusetts.

47. My correspondent in Maryland purchased a cargo of flour for me, for £437 that currency; how much Massachusetts money must I remit him, £125 Maryland being equal to £100 Massachusetts?

As £125 : £100 :: £437 : £349 12s. *Ans.*

Or, As £5 : 437 :: 4 : 349 12s. } Because £5 Maryland are equal to £4 Massachusetts.

48. A bill of exchange was accepted at Newburyport for the payment of £345 10s, for the like value delivered in Newyork, at £133½ Newyork currency for £100 Massachusetts ditto; how much money was paid in Newyork, £75 Massachusetts being equal to £100 of Newyork?

£. Mass. £. N.Y. £. Mass. £. N.Y.

As 75 : 100 :: 345 10 : 460 13 4 *Ans.*

49. When the exchange from Massachusetts to Georgia is £83½ Georgia per £100 Massachusetts, how much Massachusetts money must be paid in Boston to balance £457 Georgia currency?

£. Geor. £. Mass. £. Geor. £. Mass.

As 83½ : 100 :: 457 : 548 8s. *Ans.*

50. A merchant delivered at Boston £320 Massachusetts currency, to receive £400 in Philadelphia; what was the Massachusetts pound valued at in Philadelphia?

B.

As £320 : £400 :: £1 : £1 5s. *Answer.*

£. £. £. £.

Or, As 320 : 400 :: 1 : 1 5s. } Because £80 Massachusetts are equal to £100 Pennsylvania.

£1. 1s.

RULE OF THREE DIRECT,

81. If I draw a bill of exchange for £537 10s. 6d. Massachusetts, to be paid in Ireland, at £123 1s. 6d. Massachusetts, per £100 Irish; for how much Irish money must I draw the bill?

Mass. Irish. Mass. Irish.
 As £123 1s. 6d. : £100 :: £537 10s. 6d. : £436 14/9
 Or, as 123 1s. 6d. : 100 :: 537 10s. 6d. : 436 14/9
 { Because £16 Mass. are = £13 Irish.

52. Suppose a bill is drawn in Ireland, and payable in Boston, for £673 1s. 6d. Irish; how much Massachusetts money comes it to, the exchange at £81 1s. 6d. Irish per £100 Massachusetts?

As 81 1s. 6d. : 100 :: 673 1s. 6d. : 829 1s. 6d.

The value of any quantity of silver in any of the currencies of the United States, may be found by the following Proportion.

As the number of grains, contained in £1 is to £1; so are the grains, in any given quantity, to its value.

53. What is the value of 1lb of silver in Massachusetts currency; the pound (or 20 shillings) containing 1399 1/2 grains?

As 1399 1/2 : 1 :: 5760 : 4 2/8.

RULE OF THREE DIRECT, IN VULGAR FRACTIONS.

RULE. Having made the necessary preparations, as directed in multiplication and division of vulgar fractions, state your question as in whole numbers, and invert the first term of the proportion; then multiply the three terms continually together, and the product will be the answer.

* This Rule and the next, depend upon the same principle as the Rule of Three in whole numbers.

1. If $\frac{3}{4}$ of a yard cost $\frac{1}{2}$ of a £, what will $\frac{1}{2}$ of an ell English cost?

$$\frac{1}{2} \text{ yd.} = \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} = \frac{5 \times 4 \times 3}{8 \times 1 \times 2} = \frac{1}{2} \text{ ell English.}$$

E. Eng. £. E. Eng.

$$\text{As } \frac{1}{2} : \frac{1}{2} :: \frac{1}{2} \text{ And } \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 5 \times 3}{2 \times 1 \times 2} = \frac{15}{4} = 3 \frac{3}{4} \text{ Ans.}$$

2. If $\frac{3}{4}$ yd. cost £ $\frac{1}{2}$, what will 40 $\frac{1}{2}$ yds. come to?

$$\text{Ans. } £ 59 \text{ 11. } 3 \frac{1}{2} \text{d.}$$

3. If 70 bushels of corn cost £12 $\frac{1}{2}$; what is it per bushel?

$$\text{Ans. } 3 \text{ s. } 7 \frac{1}{2} \text{d.}$$

4. If $\frac{1}{10}$ of a ship cost £51; what are $\frac{3}{5}$ of her worth?

$$\text{Ans. } £ 10 \text{ 18. } 6 \frac{1}{2} \text{d. } \frac{3}{4}$$

5. At £3 $\frac{1}{2}$ per Cwt. what will 5 $\frac{1}{2}$ B come to?

$$\text{Ans. } 6 \text{ s. } 3 \frac{1}{2} \text{d. } \frac{5}{8}$$

6. A person having $\frac{2}{3}$ of a vessel, sells $\frac{1}{3}$ of his share for £319; what is the whole vessel worth?

$$\text{Ans. } £ 598 \text{ 11. } 6 \text{d.}$$

7. A merchant sold 51 pieces of cloth, each containing 12 $\frac{1}{2}$ yds. at 6 $\frac{1}{2}$ d. per yard; what did the whole amount to?

$$\text{Ans. } £ 31 \text{ 9. } 10 \frac{1}{2} \text{d. } \frac{1}{2}$$

8. A buys of B £560 $\frac{1}{2}$ bank stock, at £85 $\frac{1}{2}$ per cent. what comes it to?

$$\text{Ans. } £ 480 \text{ 7. } 6 \frac{1}{2} \text{d.}$$

9. A merchant makes insurance upon a vessel and cargo, valued at £3750 16s. at 15 $\frac{1}{4}$ guineas per cent. what does the premium amount to?

$$\text{Ans. } £ 818 \text{ 18. } 5 \frac{1}{2} \text{d.}$$

10. A merchant in Holland draws a bill upon his correspondent in Boston for 3750 Ducats at 1 $\frac{1}{2}$ s. 4 $\frac{1}{2}$ d. how much Massachusetts currency must he receive?

$$\text{Ans. } £ 1565 \text{ 11. } 6 \text{d.}$$

11. A gentleman from Boston being in England, where the price of silver is to that of gold, as 1 to 15 $\frac{1}{2}$, exchanged 15 $\frac{1}{2}$ lb. of silver for gold; on his return to Massachusetts, where the price of silver is to that of gold, as 1 to 15 $\frac{1}{2}$; a friend, wanting his gold, gave him the value thereof in silver; what weight of silver did he gain by the exchange?

As

R. Sil. G. R. Sil. R. Gold. G. Sil. G. R. Sil.
 As $15\frac{1}{4} : \frac{1}{4} :: 158\frac{1}{4} : 10\frac{1}{4}$. As $\frac{1}{4} : 15\frac{1}{4} :: 10\frac{1}{4} : 16\frac{1}{4}$.
 (Ans. $4\frac{1}{4}$ bales.)

12. A merchant bought a number of bales of velvet, each containing $129\frac{1}{4}$ yards, at the rate of 7 dollars for 5 yards, and sold them out at the rate of 11 dollars for 7 yards; and gained 200 dollars by the bargain; how many bales were there?

Yds. Dol. Yds. Dol. { Sold 5 yards for $7\frac{2}{7}$ Dollars.
 As $7 : 11 :: 5 : 7\frac{2}{7}$ { Bought 5 yds. for 7 Dollars.
 { In 5 yards gain $\frac{1}{7}$ Dollar.

D. Yds. D. Yds. Yds. B. Yds. Bales.
 As $\frac{5}{7} : 5 :: 200 : 1166\frac{2}{3}$. As $129\frac{1}{4} : \frac{1}{4} :: 1166\frac{2}{3} : 9$ bales.

Although the method before laid down be universally applicable, yet there are other methods more ready and expeditious in some particular cases.

RULE 1.—If the first and third terms be fractions, and the second a whole number, reduce the first and third to one common denominator; then, rejecting the denominators, make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{1}{2}$ yard cost 9s. what cost $\frac{1}{4}$ yard at that rate?
 $\frac{1}{2} = \frac{2}{4}$ & $\frac{1}{4} = \frac{1}{4}$. Now, As $15 : 9 :: 14 : 8\frac{1}{2}$ d. 4.

RULE 2.—If, of the first and third terms, one be 1, and the other a fraction; put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land cost £12; what will $\frac{1}{4}$ of an acre cost, at that rate? Den. £. Num. £. s.

As $1 : 12 :: \frac{1}{4} : 3$ £. 10 Ans.

RULE 3.—If the second term be a fraction like-wise, (that is, if all the terms be fractions;) having reduced the first and third to one common denominator, multiply the numerator of the first term by the denominator of the second, for a divisor;—and the

If $\frac{1}{2}$ yard of cloth cost $\frac{1}{4}\text{£}$. what cost $\frac{7}{8}$ yard?
 $\frac{1}{2} = \frac{2}{4}$, which reduces it to a common denominator,
 then, As 4 : $\frac{1}{4}$:: 7

$$\text{As } 4 : \frac{3}{4} :: 7$$

4 8

$$16) 21(1\frac{5}{16} = 26s. 3d. \text{ Ans.}$$

16

5

THEOREM 1. As 1 : $5\frac{1}{3}$:: 12 : 64 :: 1 : $5\frac{1}{3}$:: 1 : $2\frac{2}{3}$
(Case 1.) = $2\frac{2}{3}$ (Case 2.) = $5\frac{1}{3}$ (Case 3.) = $\frac{2}{3}$; therefore,

1. What is the value of 18 grains of gold?

$$18 \times 2\frac{2}{3} = 48d. = 4r.$$

1. What is the value of 7oz. 3pwt. 16gr. of gold?

gr. gr. oz. pwt. gr.
 $16 \div 2 = 8$, then, 7 8 8
 8

$$\begin{array}{r} 87 \\ 2 \end{array} \quad \begin{array}{r} 8 \\ 9 \end{array} \quad \begin{array}{r} 4 \\ 6\frac{2}{3} \end{array}$$

£39 12 103 Ans.

RULE 3.—If the given quantity consist of pounds only, multiply by 64, and the product will be the answer:

O

but if it consist of pounds, ounces, &c. it will be most convenient to reduce the pounds to ounces, and proceed by Rule 2.

What is the value of 36 $\frac{1}{2}$ lb of gold, at £64 per lb?
 $36 \times 64 = \text{£}2304$ Ans.

PROB. 2. To ascertain the value of any given quantity of gold in Spanish milled Dollars.

THEOREM 2. 1 $\frac{1}{2}$ wt. of gold = 5 $\frac{1}{2}$ s. 1 dol. = 6s.
 And $5\frac{1}{2} = \frac{11}{2} = \frac{2}{3}$, therefore,
 $\frac{6}{11}$

RULE.—Reduce the given quantity of gold to pennyweights; then, As the denominator is to the given quantity, so is the numerator to the answer in dollars. Or,

Divide by the denominator, and multiply the quotient by the numerator. Or,

Divide by the denominator, and subtract the quotient from the dividend; in either case, you will have the answer.

1. What is the value of 6oz. 6pwt. of gold, in Spanish dollars?

	6oz. 6pwt.		
	20		
	—		
	126 pwt.		
As 9 : 126 :: 8		Or,	Or,
8		9)126	9)126
—		—	—
9)1008			Subt. 14
—			—
		14 × 8 = 112 Ans.	
Ans. 112 dollars.			112 Ans.

PROB. 3. To ascertain the weight of gold equivalent to any given sum, currency.

RULE 1.—If the given sum be in pence, reverse Rule 1, Theor. 1, that is; As the numerator 8 is to the given sum in pence; so is the denominator 3 to the weight required in grains.

What

What weight of gold is equal to 45-?

$$\begin{array}{rcl} & d. & 32 \\ \text{As } 8 :: 48 :: 3 & & \hline & 3 & 48d. \end{array}$$

$$\begin{array}{r} 8 \overline{)144} \end{array}$$

Ans. 18 grains.

RULE 2.—If the given sum be in pounds, shillings and pence; As $5\frac{1}{2}$ is equal to $\frac{11}{2}$; therefore divide the given sum by 8, and that quotient by 2; add the two quotients together, double the last denomination, and you will have the answer.

What quantity of gold is equivalent £45 13s. 4d.?

$$\begin{array}{r} \text{oz. pwt. gr.} \\ \text{Mark the pounds, shillings } \} 8 \overline{)45 \ 13 \ 4} \\ \text{and pence, as oz. pwt. \& gr. } \} \hline \begin{array}{r} 2 \overline{)5 \ 14 \ 2} \\ 9 \ 17 \ 1 \end{array} \left. \vphantom{\begin{array}{r} 2 \overline{)5 \ 14 \ 2} \\ 9 \ 17 \ 1 \end{array}} \right\} \text{Add.} \\ \hline 8 \ 11 \ 3+3 \\ \hline \text{Oz. 8 \ 11 \ 6 } \text{Ans.} \end{array}$$

PROB. 4. To find the weight of gold equivalent to any given number of dollars.

RULE.—Divide the dollars by the numerator 8, add the quotient to the dividend, and the sum is pennyweights.

Required the weight of gold equal to 76 dollars?

$$76 \div 8 = 9\frac{1}{2} \text{ and } 76 + 9\frac{1}{2} = 85\frac{1}{2} \text{ pwt. } \text{Ans.}$$

RULE OF THREE DIRECT, IN DECIMALS.

RULE.—Having reduced your fractions to decimals, and stated your question as in whole numbers, multiply the second and third together; divide by the first, and the quotient will be the answer.

1. If

Cist. h. Cist. h. h. m.

Then, as, 6 : 1 :: 1 : 1,66 + = 1 40 Ans.

The following Questions are in the Coin of the United States.

6. If 19 yards cost 25 dol. 7 d. 5 c. ; what will 485½ yards come to ?

Yds. dol. d. c. Yds.
 As 19 : 25, 7 5 :: 485, 5

25, 75
 21775.
 30485
 21775
 8710

D. d. c. m.

19)11214,125(590,217½ Ans.

7. If 345 yards of tape cost 5 dol. 1 d. 7 c. 5 m. ; what will one yard cost ?

Yds. D. d. c. m. Yd.
 As 345 : 5, 1 7 5 :: 1

1
 d. c. m.
 345)5, 1 7 5(0 1 5 Ans.
 345
 1725
 1725

8. If I give 12 dol. 8 d. 2 c. 5 m. for 675 tops ; how many tops will 19 mills buy ?

D. d. c. m. T. d. c. m.
 As 12, 8 2 5 : 675 :: , 0 1 9

, 0 9
 6075
 675

12,825)12,825(1 top, Answer.
 12,825

RULE OF THREE INVERSE.

RECIPROCAL PROPORTION.

OR,

RULE OF THREE INVERSE,

Teaches, by having three numbers given, to find a fourth, which shall have the same proportion to the second, as the first has to the third.

Therefore, the greater the third term is, in respect to the first, the less will the fourth term be in respect to the second; or, the less the third term is in proportion to the first, the greater the fourth must be in proportion to the second; and this is called *reciprocal, inverted or indirect* Proportion.

The principal difficulty, that will embarrass the learner, will be, to distinguish when the proportion is direct, and when indirect.—This is done by an attentive consideration of the sense and tenor of the question proposed: For if thereby it appears, that when the third term of the stating is less than the first, the answer must be less than the second; or when the third is greater than the first, the answer must be greater than the second; then the proportion is direct: But, if the third be less than the first, and yet the sense of the question requires the fourth to be greater than the second; or if the third being greater than the first, the answer must be less than the second, the proportion is inverse.

RULE.—*State and reduce the terms as in the Rule of Three direct; then, multiply the first and second.

* If 4 men can do a piece of work in 12 days; in what time will 8 men do it?

As 4 men : 12 days :: 8 men : $\frac{4 \times 12}{8} = 6$ days, the answer.

And here the product of the first and second terms, that is 4 times 12, or 48, is evidently the time in which one man would perform the work; therefore, 8 men will do it in one eighth part of the time, or 6 days.

second terms together, and divide the product by the third; the quotient will be the answer, in the same denomination as the middle term was reduced into.

If there be fractions in your question, they must be stated as before directed; and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the answer.

EXAMPLES.

1. How much shalloon, that is $\frac{1}{4}$ yard wide, will line $6\frac{1}{2}$ yards of cloth, which is $1\frac{1}{2}$ yard wide?

Yd. yds. gr.

As $1\frac{1}{4} : 6\frac{1}{2} :: 3$

$$\begin{array}{r} 4 \quad 4 \\ \hline 5 \quad 27 \end{array}$$

grs. grs. grs.

As 5 : 27 :: 3

$$\begin{array}{r} 5 \\ \hline 3 \overline{) 135} \end{array}$$

$$4 \overline{) 45}$$

$11\frac{1}{4}$ yds. Ans.

The same by Vulgar Fractions.

First. $1\frac{1}{4} = \frac{5}{4}$, $6\frac{1}{2} = \frac{13}{2}$, and $3\text{ grs.} = \frac{1}{4}$, then

yd. yds. yd.

As $\frac{5}{4} : \frac{13}{2} :: \frac{1}{4}$ And $\frac{5}{4} \times \frac{13}{2} \times \frac{4}{1} = \frac{5 \times 27 \times 4}{4 \times 4 \times 3} = \frac{340}{3}$

$\frac{340}{3} = 11\frac{1}{4}$ yds. Ans.

The same by Decimal Fractions.

$1\frac{1}{4} = 1.25$, $6\frac{1}{2} = 6.75$, and $3\text{ grs.} = .75$, then

yd. yds. yd.

As 1.25 : 6.75 :: .75

$$\begin{array}{r} 1.25 \\ \hline 3375 \end{array}$$

$$\begin{array}{r} 3375 \\ 1350 \end{array}$$

$$\begin{array}{r} 675 \end{array}$$

2. What length of board

$7\frac{1}{2}$ inches wide, will make a square foot?

In. br. In. len. In. br. In. len.

As 12 : 12 :: $7\frac{1}{2}$: 19 $\frac{1}{2}$ Ans.

$19\frac{1}{2}$ or 1375 11.25 yds. Ans.

3. How

3. How many yards of carpet, $2\frac{1}{4}$ feet wide, will cover a floor, which is 18 feet long and 16 feet wide?

ft. ft. ft. ft. yds.

As 16 : 18 :: $2\frac{1}{4} \times 3$: $34\frac{1}{11}$ *Ans.*

Note, I multiply $2\frac{1}{4}$ by 3, because 3 feet = 1 yd.

4. Suppose I lend a friend £350 for 5 months, he promising the like kindness; but, when requested, can spare but £125; how long may I keep it to balance the favor?

£. Mo. £. Mo.

As 350 : 5 :: 125 : 14 *Ans.*

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain, 9 months?

Mo. Men. Mo. Men. Men.

As 5 : 450 :: 9 : 250. & 450—250 = 200 Men. *Ans.*

6. If a man performs a journey in 15 days, when the day is 12 hours long; in how many will he do it, when the day is but 10 hours?

h. d. h. d.

As 12 : 15 :: 10 : 18 *Answer.*

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre; how wide must it be, when it is but 19 rods long, to make an acre?

Leng. Br. Leng. Br. ft. in.

As 40 : 4 :: 19 : 8 6 $\frac{11}{19}$ *Ans.*

8. If, when wheat is 6s. per bushel, the two penny loaf weighs 9,6oz. what ought it to weigh, when wheat is 7s. 6d. per bushel?

s. oz. s. d. oz. pwt. gr.

As 6 : 9,6 :: 7 6 : 7 13 14,4 *Answer.*

9. If a piece of board be 30 inches in length; what breadth will make $1\frac{1}{4}$ square foot?

ft. ft. in. in. in.

As 1,5 : 1 :: 30 : 7,2 *Answer.*

10. If

RULE OF THREE INVERSE. 163

10. If 9 men can build a house in 5 months, by working 14 hours per day; in what time will the same number of men do it, when they work only 10 hours per day?

h. mo. h. mo.

As 14 : 5 :: 10 : 7 *Answer.*

11. A wall which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days; how many men must be employed, to finish the wall in 4 days?

ft. men. ft. men.

As 8 : 6 :: 24 - 8 : 12 to finish it in 12 days. And
days. men. days. men.

As 12 : 12 :: 4 : 36 to finish it in 4 days.

12. There is a cistern having a pipe, which will empty it in 6 hours; how many pipes, of the same capacity, will empty it in 20 minutes?

ho. pi. min. pi.

As 6 : 1 :: 20 : 120 *Answer.*

13. What number of men must be employed, to finish in 9 days, what 15 men would be 30 days about?

da. men. da. men.

As 30 : 15 :: 9 : 50 *Answer.*

14. If a field will feed 6 cows 91 days; how long will it feed 21 cows?

cows. d. cows. d.

As 6 : 91 :: 21 : 26 *Answer.*

15. How much in length, that is 8 $\frac{1}{2}$ inches broad, will make a foot square?

in. in. in. in.

As $\frac{144}{1}$: $\frac{1}{2}$:: 8 $\frac{1}{2}$: 16 $\frac{32}{1}$ *Answer.*

16. How much in length, that is 13 $\frac{1}{2}$ poles in breadth, will make a square acre?

po. po. po. po.

As $\frac{160}{1}$: $\frac{1}{2}$:: 13 $\frac{1}{2}$: 11 $\frac{11}{11}$ *Answer.*

17. A

17. A regiment of soldiers, consisting of 745 men, is to be clothed, each suit, to contain $3\frac{1}{2}$ yards of cloth, which is $\frac{3}{8}$ yard wide, and lined with shalloon $\frac{7}{8}$ yard wide; how many yards of shalloon will line them?

As $745 \times 3\frac{1}{2} : 1\frac{3}{8} :: \frac{7}{8} : 4097\frac{1}{2}$ yds. *Ans.*

18. If a suit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating, $\frac{7}{8}$ of a yard wide, will it require for the same person?

yd. yds. yd. yds. qr. n.

As $1\frac{3}{8} : 4\frac{1}{8} :: \frac{7}{8} : 6 \ 1 \ 3\frac{5}{7}$ *Answer.*

ABBREVIATIONS.

To know whether a fraction, when abbreviated, be equivalent in all respects to the original given fraction.

RULE.—As the numerator of the fraction, in its lowest terms, is to its denominator; so will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other; so will one denominator be to the other, &c.

A owes B £75 13s. 6d. now £100 of A's money is equal to £140 of B's; what must A pay to satisfy the said debt?

$$\frac{100}{140} = \frac{5}{7} \text{ therefore, } \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 75 \quad 13 \quad 6 \\ \underline{\hspace{1cm}} \end{array}$$

$$\begin{array}{r} 7)378 \quad 7 \quad 6 \\ \underline{\hspace{1cm}} \end{array}$$

$$\text{£}54 \quad 1 \quad 0\frac{6}{7} \text{ *Ans.*}$$

Now, to prove whether $\frac{5}{7}$ be equal to $\frac{100}{140}$.

Num. Den. Num. Den. Num. Num. Den. Den.

As 5 : 7 :: 100 : 140—Or, as 5 : 100 :: 7 : 140.

COMPOUND.

COMPOUND PROPORTION,

OR,

DOUBLE RULE OF THREE,

Teaches to resolve such questions as require two, or more statings, by simple proportion; and that, whether direct, or inverse:—It is composed (commonly) of 5 numbers to find a sixth, which, if the proportion be direct, must bear such proportion to the 4th and 5th, as the 3d bears to the 1st and 2d: But if inverse, the 6th number must bear such proportion to the 4th and 5th, as the 1st bears to the 2d and 3d.

Always place the three conditional terms in this order: That number, which is the principal cause of gain, loss or action, possesses the first place; that which denotes the space of time or distance of place, the second; and that, which is the gain, loss or action, the third; this being done, place the other two terms, which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion; therefore,

RULE 1.—Multiply the three last terms together, for a dividend, and the two first for a divisor:—But if the blank fall under the first or second place; then, the proportion is inverse; therefore,

RULE 2.—Multiply the first, second and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

1. If £100 gain £6 in a year; what will £400 gain in 9 months?

£. P. Mo. £. Int.

100 : 12 :: 6 Terms in the supposition, or conditional terms.

400 : 9 Terms which move the question.

Here, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first rule; therefore,

COMPOUND PROPORTION.

$$400 \times 9 \times 6 = 21600 \text{ For the dividend, and}$$

$$100 \times 12 = 1200 \text{ For the divisor.}$$

See the work at large.

£.Pr. Mo. £.Int.

$$100 : 12 :: 6$$

$$400 : 9$$

9

$$12,00 \overline{) 216,00} (18 \text{ £. Answer.}$$

12

$$100 \quad 3600$$

$$12 \quad 6$$

$$12$$

$$96$$

$$96$$

$$12,00 \overline{) 216,00} ($$

2. If £100 will gain £6 in a year; in what time will £400 gain £18?

$$100 : 12 :: 6 \text{ Terms in the supposition.}$$

$$400 : \quad :: 18 \text{ Terms which move the question.}$$

Here, the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second Rule: Therefore,

$$100 \times 12 \times 18 = 21600 \text{ For the dividend.}$$

$$400 \times 6 = 2400 \text{ For the divisor.}$$

£.Pr. mo. £.Int.

$$100 : 12 :: 6$$

$$400 \quad :: 18$$

6

12

$$2400$$

$$216$$

$$100$$

$$24 \overline{) 216} (9 \text{ months, Answer.}$$

216

3. If £400 gain £18 in 9 months; what is the rate per cent. per annum?

Pr.

COMPOUND PROPORTION. 259

Pr. mo. Int.

400 : 9 :: 18

100 : 12 :: 18

18

96

12

400

216

100

216

216

216

4. What principal, at 6 per cent. per ann. will gain £18 in 9 months?

Pr. mo. Int.

400 : 12 :: 6

9 : 18 :: 18

18

9

216

6

100

216

216

5. If 8 men spend £32 in 13 weeks; what will 24 men spend in 52 weeks?

M. W. £.

8 : 13 :: 32

24 : 52 :: 32

32

24

52

32

32

32

32

32

32

32

32

32

32

32

6. If the freight of 9 hhds. of sugar, each weighing 12 Cwt. at 100 leagues, cost £16; what must be paid for the freight of 50 tierces ditto, each weighing 2½ Cwt. 100 leagues?

Hhds. Wt. £.

9 : 100 :: 16

50 : 100 :: 16

16

50

100

16

50

100

16

50

100

16

50

100

7. There

170 COMPARISON OF WEIGHTS, &c.

7. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such an one should be built in 5 months; I demand the number of men to be employed about it?

Men. mo. Ed.

$$20 : 12 :: 1$$

$$5 :: 1 \quad 48 \text{ men, } \textit{Ans.}$$

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days; in what time will 24 men build one 200 feet long, 3 feet high and 6 feet thick?

Men. da. ft.

$$6 : 16 :: 20 \times 6 \times 4$$

$$24 : :: 200 \times 3 \times 6 \quad 20 \text{ days, } \textit{Ans.}$$

COMPARISON OF WEIGHTS and MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown; how many Massachusetts pence are worth 320 French crowns?

Fr. Cr. d. Fr. Cr.

$$\text{As } 1 : 78 :: 320 : 24960. \textit{Ans.}$$

2. If 24 yards at Boston make 16 ells at Paris, how many ells at Paris will make 128 yards at Boston?

Bost.

Par.

Bost.

Par.

$$\text{As } 24 \text{ yds.} : 16 \text{ ells.} :: 128 \text{ yds.} : 85 \text{ ells. } \textit{Ans.}$$

3. If 60 lb at Boston make 56 lb at Amsterdam; how many lb at Boston will be equal to 350 at Amsterdam?

Ans. Bost.

Ans. Bost.

lb

lb

$$\text{As } 56 : 60 :: 350 : 375 \textit{ Ans.}$$

CONJOINED

CONJOINED PROPORTION

Is when the coins, weights or measures of several countries are compared in the same question ; or, in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This Rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble—and it may be proved by as many statings in the single Rule of Three as the nature of the question may require.

C A S E I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

RULE.—Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand ; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

E X A M P L E S.

1. Suppose 100 yards of America = 100 yards of England, and 100 yards of England = 50 canes of Thoulouse, and 100 canes of Thoulouse = 160 ells of Geneva, and 100 ells of Geneva = 200 ells of Hamburgh ; how many yards of America are equal to 379 ells of Hamburgh ?

Antecedents;

<i>Antecedents.</i>	<i>Consequents.</i>	<i>Abridged.</i>
100 of America =	100 of England.	<i>Ant. Con.</i>
100 of England =	50 of Thoulouse.	5 8
100 of Thoulouse =	160 of Geneva.	879
100 of Geneva =	200 of Hamburg.	
379 of Hamburg ?	Therefore,	
$\frac{379 \times 5}{8} = 236\frac{1}{2} \text{ yds. of America} = 379 \text{ ells of Hamburg.}$		

ILLUSTRATION.

The two 100's on both sides cancel each other : Let the last cyphers of the three next antecedents and consequents be cancelled, which is dividing by 10 ; then divide the second antecedent and consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents ; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. — 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2 : Now, there being 2 left on each side, they cancel each other, and as there is no farther room for abridging, by reason of the odd number 379, the operation is finished, and the answer found, as above.

2. If 20*lb* at Boston make 23*lb* at Antwerp, and 155 at Antwerp make 180 at Leghorn ; how many at Boston are equal to 144 at Leghorn ?

Antecedents. Consequents.

<i>lb</i>	<i>lb</i>	
20 of Boston =	23 of Antwerp.	$20 \times 155 \times 144 = 46400 \text{ Div.}$
155 of Antwerp =	180 of Leghorn.	$23 \times 180 = 4140 \text{ Divisor.}$
144 of Leghorn.		$46400 \div 4140 = 107\frac{1}{3} \text{ Ans.}$

Or, abridged, $\frac{155 \times 144}{23 \times 9} = 107\frac{1}{3}$.

3. If 12*lb* at Boston make 10 at Amsterdam, 100*lb* at Amsterdam, 120 at Paris ; how many *lb* at Boston are equal to 80*lb* at Paris. *Ans.* 80*lb*.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American

ARBITRATION OF EXCHANGES. 175

American yards : how many Venetian braces are equal to 34 American yards ? *Ans.* 52 $\frac{1}{2}$.

5. If 40lb at Newburyport make 36 at Amsterdam, and 90lb at Amsterdam make 116 at Dantzick ; how many lb at Newburyport are equal to 260lb at Dantzick ? *Ans.* 224 $\frac{1}{2}$.

C A S E II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

RULE.—Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand ; then multiply the first row for a divisor, and the second for a dividend.

E X A M P L E S.

1. If 12lb at Boston make 10lb at Amsterdam, and 100lb at Amsterdam 120 at Paris ; how many at Paris are equal to 80 at Boston ?

Left. Right.

Boston 12 10 $10 \times 120 \times 80 = 96000$
 Amsterdam 100 120 $\frac{96000}{100 \times 120} = 80$ *Ans.*
 80 12 $12 \times 100 = 1200$

2. If 40lb at Newburyport make 36 at Amsterdam, and 90lb at Amsterdam make 116 at Dantzick ; how many lb at Dantzick are equal to 244 at Newburyport ? *Ans.* 283 $\frac{1}{2}$.

ARBITRATION OF EXCHANGES.

By this term is understood how to choose, or determine the best way of remitting money from a broad with advantage ; which is performed by conjoined proportion : Thus,

Suppose, a merchant has effects at Amsterdam to the amount of 3530 dollars, which he can remit by way of Lisbon at 840 rees per dollar, and thence to Boston,

174 SINGLE FELLOWSHIP.

Boston, at 8s. 1d. per milree (or 1000 rees :)—Or, by way of Nantz, at $5\frac{2}{3}$ livres per dollar, and thence to Boston at 8s. 8d. per crown: It is required to arbitrate these exchanges, that is, to choose that which is most advantageous?

1 Dollar at Amsterdam = 840 rees at Lisbon.

1000 rees at Lisbon. = 97d. at Boston.

3530 Dollars at Amsterdam.

$\frac{840 \times 97 \times 3530}{1000 \times 1} = \pounds 1198 \text{ 8s. } 8\frac{4}{10}d.$ By way of Lisbon.

1 Dollar at Amst. = $5\frac{2}{3}$ livres at Nantz.

6 Livres at Nantz = 80 pence at Boston.

3530 dollars at Amsterdam.

$\frac{5\frac{2}{3} \times 80 \times 3530}{1 \times 6} = \pounds 1059$ By way of Nantz.

Here it may be observed, that the difference is $\pounds 139 \text{ 8s. } 8\frac{4}{10}d.$ in favour of remitting by way of Lisbon rather than by Nantz, which depends on the course of exchange, at that time; but the course may vary so, that in a short time, by way of Nantz may be better; hence appears the necessity and advantage of an extensive correspondence, to acquire a thorough knowledge in the courses of exchange, to make this kind of remittance.

FELLOWSHIP.

The Rules of Fellowship are those by which the accounts of several merchants, or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the joint stock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP.

Is, when the stocks are employed for any certain equal time.

RULE.

SINGLE FELLOWSHIP.

175

RULE.—As the whole stock is to the whole gain or loss; so is each man's particular stock, to his particular share of the gain or loss.

PROOF. Add all the particular shares of the gain or loss together, and, if it be right, the sum will be equal to the whole gain or loss.

EXAMPLES.

1. Divide the number 360 into 4 such parts, which shall be to each other as 3, 4, 5, & 6.

$$\text{As } 3+4+5+6 = 360 :: \left\{ \begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array} \right\} \text{ Answer.}$$

Proof.

2. A, B, C and D companied;—A put in £145; B 219; C 378, and D 417, with which they gained £569; what was the share of each?

	Whole Stock.	Gain.	Share.
A	145	71	3 8 11 11 11 A's
B	219	107	10 3 1 11 11 B's
C	378	185	3 11 11 11 11 C's
D	417	204	14 5 1 11 11 D's

£569 — — — *Proof.*

3. A, B, C, and D are concerned in a joint stock, of £168 2s. 6d.; of which A's part is £36 10s. B's £37 15s. C's £49, and D's £55 17s. 6d.: Upon the adjustment of their accounts, they have lost £23 13s. 4d.: What is the loss of each?

Ans. A's loss £11 8s. 5 1/2d. B's £16 10s. 9 1/2d. C's £21 9s. 4 1/2d. and D's £24 9s. 7 1/2d.

4. A

* That their gain or loss, in this rule, is in proportion to their stocks, is evident: For, as the times, in which the stocks are in trade, are equal, if I put in 1/2 the whole stock, I ought to have 1/2 the gain.

4. A and B companied : A put in £45, and took $\frac{1}{3}$ of the gain : What did B put in ?

Ans. $6-8=2$. Then, as $3:45::2:30$ *Ans.*

5. A, B and C freighted a ship, with 68900 feet of boards : A put in 16500 feet ; B 28750 ; and C the rest : But, in a storm, the Captain threw overboard 26450 feet : How much must each sustain of the loss ?

Ans. A, 6841 $\frac{1}{2}$ feet, B 11036 $\frac{1}{2}$, and C 9071 $\frac{1}{2}$ feet.

6. A gentleman died, leaving three sons and a daughter ; to whom he bequeathed his estate in the following manner : To the eldest son, he gave 312 moidores ; to the second, 312 guineas ; to the third, 312 pistoles ; and to the daughter, 312 dollars ; but when his debts were paid, there were but 312 half joes left : What must each have in proportion to the legacies which had been bequeathed them ?

Ans. 1st son £293 or. 2s. — 2d son £227 17s. 10 $\frac{1}{2}$ d. — 3d son £179 17s. 2 $\frac{1}{2}$ d. — and the daughter £48 16s. 8 $\frac{1}{2}$ d.

7. A ship worth £780, being lost at sea, of which $\frac{1}{3}$ belonged to A, $\frac{1}{4}$ to B, and the rest to C ; what loss will each sustain, supposing £450 to have been insured upon her ?

$780-450=330$; then $\frac{1}{3}330=110$; $\frac{1}{4}330=82\frac{1}{2}$; $\frac{1}{2}330=165$.

$£55=A's$ $£165=B's$ $£110=C's$ share.

8. A and B venturing equal sums of money, cleared by joint trade £140 : By agreement, as A executed the business, he was to have 8 per cent., and B was to have 5 per cent. What was A allowed for his trouble ?

$£. £. £. £. £. £. £. £. £.$
As $8+5:140::8:86\frac{2}{3}$ & As $8+5:140::5:51\frac{1}{3}$

Ans. £33 6s. 1 $\frac{2}{3}$ d.

9. A bankrupt is indebted to A £120, to B 280, to C 340, and to D 450, and his whole estate amounts only to £560 ; how must it be divided among the creditors ?

Ans. A £58 18s. 11½d. B £112 19s. 7½. C £167 0s. 4d. D £221 1s. 0½d.

10. A, B and C put their money into a joint stock : A put in £40, B and C together, 170 ; they gained £126, of which B took 42 ; what did A and C gain, and B and C put in respectively ?

As 210 the whole stock : 126 the whole gain :: 40 A's stock : 24 A's gain.

As £24 A's gain : 40 A's stock :: 42 B's gain : 70 B's stock. Then £170 - £70 = £100 C's stock ; and the whole gain £126 - £66 A's and B's gain = £60 C's gain.

11. A, B, and C companied : A put in £40 ; B £60, and C a sum unknown : They gained £72 ; of which C took 32 for his share : What did A and B gain, and C put in ?

The whole gain £72 - C's gain £32 = £40 A's and B's gain : Then, As £100, A's and B's stock : £40 their gain :: £40 A's stock : £16, his gain. Again, As £16 A's gain : £40, his stock :: £32, C's gain : £80 his stock.

12. A, B and C put in £720, and gained £540, of which, so often as A took up £3, B took up 5, and C 7 ; what did each put in, and gain ?

Money taken up. { 3 : 108 A's gain.

As 3+5+7 : 540 :: { 5 : 180 B's ditto.

{ 7 : 252 C's ditto.

And, as 3+5+7 : 720 :: { 3 : 144 A's stock.

{ 5 : 240 B's ditto.

{ 7 : 336 C's ditto.

Or, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720 48 multiplicand, to find the stocks—and 15)540 36 multiplicand, to find the gains.

£. £.
48×3=144 A's stock. } { 36×3=108 A's gain.
48×5=240 B's ditto. } { 36×5=180 B's ditto.
48×7=336 C's ditto. } { 36×7=252 C's ditto.
as before.

13. A, B, C and D companied, and gained a sum of money; of which, A, B and C took £120, B, C and D, £180, C, D and A, £160, and D, A and B, £140; what distinct gain had each?

The sum of these 4 numbers is £600, and as each man's money is named 3 times, therefore $\frac{1}{3}$ of £600 is the whole gain—Therefore £200—120 A's, B's and C's gain = 80 D's gain :—And 200—180 B's, C's and D's gain = 20 A's gain. 200—160 C's, D's and A's gain = 40 B's gain. And £200—140 D's, A's and B's gain = 60 C's gain.

14. Two merchants companied : A put in £40, and B 288 ducats : They gained £135, of which A took £60; what was the value of a ducat?

As 60, A's gain : 40, his stock :: 135, the whole gain—60, A's gain : 50, B's gain.

Duc. £. Duc. s. d.

And, As 288 : 50 :: 1 : 3 $\frac{5}{8}$ Ans.

15. Four men spent, at a reckoning, 20 shillings; of which they agreed that A should pay $\frac{1}{2}$, B, $\frac{1}{4}$, C, $\frac{1}{8}$, and D, $\frac{1}{16}$; what must each pay in that proportion?

	s. d.
Shares to be paid s.	15 : 0 $\frac{21}{16}$
As $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} : 20 ::$	10 : 6 $\frac{11}{16}$
	5 : 3 $\frac{0}{16}$
	2 6 : 1 6 $\frac{11}{16}$

DOUBLE FELLOWSHIP.

Or, *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

RULE.—Multiply each man's stock, or share, by the time it was continued in trade; Then,

As the whole sum of the products, is to the whole gain or loss :

So

When the times are equal, the shares of the gain or loss are equally as the stocks, as in Single Fellowship; and when the stocks are equal, the shares are as the times; wherefore, when neither are equal, the shares must be as their products.

DOUBLE FELLOWSHIP.

179

So is each man's particular product, to his particular share of the gain or loss.

1. A, B and C hold a pasture in common, for which they pay £40 per annum. A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C, 8 oxen for 16 weeks; what must each pay of the rent?

$9 \times 5 = 45$, $12 \times 7 = 84$, and $8 \times 16 = 128$; then $45 + 84 + 128 = 257$.

As 257 : 40 :: 45 : 6s 3d; 84 : 10s 6d; 128 : 20s 0d

45	84	128
200	160	257
160	320	514
257	514	1028

257)1800(7	257)8360(32	257)1028(4
1799	8360	1028
11	794	80
30	377	257

257)1000(3	257)1920(7	257)1028(4
771	1920	1028
229	1663	80

257)1400(5	257)3800(14	257)1028(4
1285	3800	1028
115	3672	80
257)960(3	257)1140(4	257)1028(4
771	1140	1028
189	114	80

257)1400(5	257)3800(14	257)1028(4
1285	3800	1028
115	3672	80

2. Four merchants traded in company, A put in £100, for five months; B, 150 for 6 months; C, 210, for 8 months; and D, 310, for 9 months; but

13. A, B, C and D companied, and gained a sum of money; of which, A, B and C took £120, B, C and D, £180, C, D and A, £160, and D, A and B, £140; what distinct gain had each?

The sum of these 4 numbers is £600, and as each man's money is named 3 times, therefore $\frac{1}{3}$, viz. £200 is the whole gain—Therefore £200—120 A's, B's and C's gain = 80 D's gain :—And 200—180 B's, C's and D's gain = 20 A's gain. 200—160 C's, D's and A's gain = 40 B's gain. And £200—140 D's, A's and B's gain = 60 C's gain.

14. Two merchants companied : A put in £40, and B 288 ducats : They gained £135, of which A took £60; what was the value of a ducat?

As 60, A's gain : 40, his stock :: 135, the whole gain—60, A's gain : 50, B's gain.

Duc. £. Duc. s. d.

And, As 288 : 50 :: 1 : 3 $\frac{5}{8}$ Ans.

15. Four men spent, at a reckoning, 20 shillings; of which they agreed that A should pay $\frac{1}{2}$, B, $\frac{1}{4}$, C, $\frac{1}{8}$, and D, $\frac{1}{16}$; what must each pay in that proportion?

	s. d.	s. d.
Shares to be paid s.	15	9 $\frac{2}{3}$
As $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} : 20 ::$	10	6 $\frac{1}{2}$
	5	3 $\frac{1}{4}$
	2 6	1 6 $\frac{1}{8}$

Ans.

DOUBLE FELLOWSHIP.

Or, *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

RULE.—Multiply each man's Stock, or share, by the time it was continued in trade : Then,

As the whole sum of the products, is to the whole gain or loss :

So

* When the times are equal, the shares of the gain or loss are evidently as the Stocks, as in Single Fellowship : and when the Stocks are equal, the shares are as the times ; wherefore, when neither are equal, the shares must be as their products.

DOUBLE FELLOWSHIP.

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So is each man's particular product, to his particular share of the gain or loss.

1. A, B and C hold a pasture in common, for which they pay £40 per annum. A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C, 8 oxen for 16 weeks; what must each pay of the rent?

$9 \times 5 = 45$, $12 \times 7 = 84$, and $8 \times 16 = 128$; then $128 + 84 + 45 = 257$.

As $257 : 40 :: 45 : 6s 2d$, $84 : 10s 6d$, $128 : 16s 8d$.

$\frac{45}{200}$ $\frac{84}{160}$ $\frac{128}{320}$

$257)1800(7$ $257)3360(13$ $257)5120(19$

$\frac{1799}{20}$ $\frac{257}{798}$ $\frac{257}{80}$

$237)2000$ $\frac{19}{20}$ $257)4740(18$

$\frac{12}{257)3800(14$ $\frac{257}{2056}$

$257)960(3$ $\frac{114}{12}$

$\frac{771}{189}$ $257)476(5$ $257)1368(5$

$\frac{257}{1285}$ $\frac{257}{1285}$

$257)764(2$ $257)333(1$

$\frac{514}{257}$ $\frac{257}{257}$

$257)1000(3$ $257)1000(3$

$\frac{771}{229}$ $\frac{257}{229}$

2. Four merchants traded in company, A put in £100, for five months; B, 150 for 4 months; C, 210, for 8 months; and D, 310, for 6 months;

but

but by misfortunes at sea they lost £145: what must each man sustain of the loss?

Ans. } A, £11 17/8 $\frac{11}{8}$ C, £41 16/8 $\frac{11}{8}$
 } B, 24 19/2 $\frac{11}{8}$ D, 66 6/4 $\frac{11}{8}$

3. A, with a capital of £100 began trade, January 1st, 1792, and meeting with success in his business, he took in B as a partner, on the first day of March following, with a capital of £150. Three months after that, they admit C, as a third partner, who brought into stock £180, and after trading together till the 1st of January, 1793, they found there had been gained since A's commencing business, £177 13s. How must this be divided among the partners?

Ans. A, £53 16/8—B, 67 5/10—C, £56 10/6.

4. Two merchants entered into partnership for 18 months; A, at first, put into stock, £100, and at the end of 8 months, he put in 50 more; B, at first, put in £275, and at 4 month's end, took out £70. Now, at the expiration of the time, they found they had gained £263: What is each man's just share?

Ans. A, £96 9/6. B, £166 10/6.

5. A and B companied; A put in the 1st of January £150; but B could not put in any till the 1st of May: What did he then put in, to have an equal share with A, at the year's end?

M. £. M.

As 12 : 150 :: 8 : $\frac{160 \times 8}{8} = £225$ *Ans.*

6. A, B and C companied; A put in, the 1st of March, £30; B, the first of May, put in 80 yards of broadcloth; and on the first of June, C put in 120 dollars. On the first of January following, they reckoned their gains, of which A and B took £228; B and C, £215 10s. and C and A, £187 10s. What was the whole gain, and the gain of each? What did they value a yard of cloth at? What was C's dollar worth?

$28 + 215 \text{ 10s.} + 187 \text{ 10s.} = £621$, and $621 \div 2 = 310 \text{ 10s.}$ the whole gain; then, $£315 \text{ 10s.} - 238 = £87 \text{ 10s.}$ C's gain, $£315 \text{ 10s.} - £215 \text{ 10s.} = £100$ A's gain, and $£315 \text{ 10s.} - £187 \text{ 10s.} = £128$ B's gain. To find the value of one yard of cloth; say, As $£100$ A's gain : 30 his stock :: 128 B's gain : $£38 \text{ 8s.}$ then, inversely, As 10 months : $£38 \text{ 8s.}$:: 8 months : $£48$ the value of the whole cloth. As 80 yds. : $£48$:: 1 yd. : 12s. answer. Now, to find the value of a dollar; As $£100$ A's gain : 30 his stock :: $£87 \text{ 10s.}$ C's gain : $£26 \text{ 5s.}$ then, inversely, As 10 months : $£26 \text{ 5s.}$:: 7 months : $£37 \text{ 10s.}$ = 120 dollars. Lastly, As 120 dollars : $£37 \text{ 10s.}$:: 1 dollar : 6/3 answer.

FELLOWSHIP BY DECIMALS.

RULE.—Divide the whole gain, or loss, by the whole stock; and the quotient multiplied severally by each man's stock, will give the gain, or loss, of each.

EXAMPLES.

1. A, B and C companied; A put in $£200$, B, 150, and C, 50, with which they gained 800; what is the share of each?

Ans. $800 \div 400 = 2$; then, $200 \times 2 = £400 =$ A's gain, $150 \times 2 = £300 =$ B's gain, and $50 \times 2 = 100 =$ C's gain.

2. A, B, C and D trade, and gain $£200$, which is to be divided in the following manner, viz. So often as A has 6, B must have 10, C, 14, and D, 20; what is the share of each?

$6 + 10 + 14 + 20 = 50$, and $200 \div 50 = 4$, quotient; then $6 \times 4 = £24$ A's gain; $10 \times 4 = £40$ B's gain; $14 \times 4 = £56$ C's; and $20 \times 4 = £80$ D's gain.

Q

3.A,

3. A, B and C compained ; A put in £40 5s. B, £80 10s. and C, £161, with which they gained £120 ; what is each man's share of the gain ?

A's stock = 40,25

B's ditto = 80,5

C's ditto = 161

Sum total = 281,75)120,000000(,4259+

4259	4259	4259
40,25	80,5	161
21295	21295	4259
3518	340720	28554
170366		4259
	£34,28495	
£17,142475	20	£68,5699
20		20
	5,69900	
3,849500	12	11,3980
12		12
	8,388	
10,194000	4	4776
4		4
	1,552	
0,776000		3,104

Proof. A's gain £17 2s. 10d. + B's gain £34 5s. 8½d. + C's gain £68 11s. 4½d. = £119 19s. 11d.

PRACTICE

Is a contraction of the Rule of Three direct, when the first term happens to be an unit, or one ; and has its name from its daily use among Merchants

chants and Tradesmen, being an easy and concise method of working most questions which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

Before the questions, hereafter given, can be wrought, the following tables must be perfectly gotten by heart.

TABLES.

* GENERAL RULE.

1. Suppose the price of the given quantity to be £1 or 1s. &c. then will the quantity itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each will be the true answer required.

EXAMPLE.

What is the value of 468 yards, at $2/6\frac{1}{2}$ per yard?

$\pounds$ 468 s. d.	Answer at £1 s. d.
2s. 6d. is $\frac{1}{2}$ = 58 10 —	ditto at — 2 6
3d. is $\frac{1}{40}$ = 5 17 —	ditto at — 3
$\frac{1}{2}$ d. is $\frac{1}{12}$ = — 9 9	ditto at — — 4
The full price = <u>£64 16 9</u>	<u>— 2 9$\frac{1}{2}$</u>

In this example, it is plain that the quantity 468 is the answer at £1; consequently, as $2/6$ is $\frac{1}{2}$ of a pound, $\frac{1}{2}$ part of that quantity, or £58 10s. is the price at $2/6$; in like manner, as 3d. is the $\frac{1}{40}$ part of 2s. 6d. so $\frac{1}{40}$ part of £58 10s. or £5 17s. is the answer at 3d. and as $\frac{1}{2}$ d. is $\frac{1}{12}$ of 3d. so $\frac{1}{12}$ of £5 17s. or 9s. 9d. is the answer at $\frac{1}{2}$ d. Now, as the sum of all these parts is equal to the whole price (2s. 9 $\frac{1}{2}$ d.) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

T A B L E S.

Aliquot, or even parts of MONEY.

Aliquot, or even parts of WEIGHT.

Pts. of a Shil. of a £.			Pts. of a pound.			Parts of a Cwt.			Pts. of a Ton.		
l.	s.	d.	s.	d.	f.	Qrs.	lb.	Cwt.	Cwt. gr.	T.	
5	—	$\frac{1}{2}$	10	0	is $\frac{1}{2}$	2	0	is $\frac{1}{2}$	10	0	is $\frac{1}{2}$
$\frac{1}{2}$	—	$\frac{1}{4}$	6	8	— $\frac{1}{3}$	1	0	— $\frac{1}{4}$	5	0	— $\frac{1}{4}$
$\frac{3}{4}$	—	$\frac{3}{8}$	5	0	— $\frac{1}{4}$	0	16	— $\frac{1}{8}$	4	0	— $\frac{1}{8}$
2	—	$\frac{1}{2}$	4	0	— $\frac{1}{5}$	0	14	— $\frac{1}{7}$	2	2	— $\frac{1}{8}$
$\frac{1}{4}$	—	$\frac{1}{8}$	3	4	— $\frac{1}{6}$	0	8	— $\frac{1}{4}$	2	0	— $\frac{1}{16}$
1	—	$\frac{1}{2}$	3	6	— $\frac{1}{8}$	0	7	— $\frac{1}{16}$	1	1	— $\frac{1}{16}$
$\frac{1}{2}$	—	$\frac{1}{4}$	1	8	— $\frac{1}{12}$	0	4	— $\frac{1}{16}$	1	0	— $\frac{1}{20}$
$\frac{3}{4}$	—	$\frac{3}{8}$	1	4	— $\frac{1}{15}$	Pts. of $\frac{1}{2}$ Cwt.			Parts of 240.		
$\frac{1}{4}$	—	$\frac{1}{8}$	1	3	— $\frac{1}{18}$	lb.	$\frac{1}{2}$ Cwt.				
$\frac{1}{8}$	—	$\frac{1}{16}$	1	0	— $\frac{1}{20}$	28	is $\frac{1}{2}$		180	is $\frac{3}{4}$	
			0	10	— $\frac{1}{24}$	14	— $\frac{1}{4}$		120	— $\frac{1}{2}$	
			0	8	— $\frac{1}{30}$	8	— $\frac{1}{8}$		80	— $\frac{1}{3}$	
			0	5	— $\frac{1}{40}$	7	— $\frac{1}{10}$		60	— $\frac{1}{4}$	
			0	2	— $\frac{1}{50}$	4	— $\frac{1}{12}$		40	— $\frac{1}{6}$	
						Pts. of $\frac{1}{4}$ Cwt.			Parts of 480.		
						lb.	$\frac{1}{4}$ Cwt.				
						14	is $\frac{1}{2}$		360	is $\frac{3}{4}$	
						7	— $\frac{1}{4}$		160	— $\frac{1}{3}$	
						4	— $\frac{1}{7}$				
						2	— $\frac{1}{14}$				

Another TABLE of Aliquot parts of Money.

Parts of a Shilling.			Parts of a Pound.			Parts of a Pound.		
d.	s.	d.	s.	d.	f.	s.	d.	f.
10	is	$\frac{5}{6}$	18	0	is $\frac{9}{10}$	13	4	is $\frac{2}{3}$
9	—	$\frac{1}{2}$	17	6	— $\frac{7}{10}$	12	6	— $\frac{2}{5}$
8	—	$\frac{2}{3}$	16	8	— $\frac{4}{5}$	12	0	— $\frac{1}{5}$
$7\frac{1}{2}$	—	$\frac{3}{4}$	16	0	— $\frac{8}{10}$	8	0	— $\frac{1}{10}$
$4\frac{1}{2}$	—	$\frac{3}{8}$	15	0	— $\frac{3}{4}$	7	6	— $\frac{3}{20}$
			14	0	— $\frac{7}{10}$	6	0	— $\frac{1}{10}$

A TABLE of DISCOUNT per cent.

£. s. d.			£. s. d.			£. s. d.		
per cent.	is	on the pound.	per cent.	is	on the pound.	per cent.	is	on the pound.
1	—	0 3	8	—	1 9	2	—	4 6
$\frac{1}{2}$	—	0 6	10	—	2 0	25	—	5 0
$\frac{3}{4}$	—	0 9	$12\frac{1}{2}$	—	2 6	30	—	6 0
5	—	1 0	15	—	3 0	33	—	7 0
$6\frac{1}{4}$	—	1 3	$17\frac{1}{2}$	—	3 6	40	—	8 0
7	—	1 6	20	—	4 0	45	—	9 0
						50	—	10 0

C A S E I.

When the price of 1 yard, 1 lb, &c. is an even part of one shilling :—Find the value of the given quantity at 1s. per yard, lb, &c. then draw a line underneath, and divide it by that even part, and the quotient will be the answer in shillings ; which must always be brought into pounds.

E X A M P L E S.

1. What will $354\frac{1}{2}$ yards cost, at $\frac{1}{4}$ d. per yard?

$\frac{1}{4}$ d. | $\frac{1}{2}$ | 354 6 value of $354\frac{1}{2}$ yards, at 1s. per yard.

Ans. £0 7 4½ value of $354\frac{1}{2}$ yards, at $\frac{1}{4}$ d. per yard.

Or thus, Or, divide by 8 and 6, thus; 8)354 6

£. s. d. s. d.
8)17 14 6 = 354 6

6)44 3½

6)2 4 3½

Ans. 7 4½ as
[before.]

7 4½ Ans. as before.

2. What will $759\frac{1}{2}$ yards come to, at 3d. per yard?

3d. | $\frac{1}{2}$ | 759 9 value at 1s. per yard.

2)0)18)9 11½

Ans. £9 9 11½ value at 3d. per yard.

Or, thus :

£. s. d.
13d. | $\frac{1}{2}$ | 37 19 9 value at 1s. per yard.

Ans. £9 9 11½ value $759\frac{1}{2}$ yds. at 3d. per yd.

Questions.

Questions. Answers.

Yds.	£.	s.	d.
3d. 642 at $\frac{1}{2}$ d. per yd.	0	13	$4\frac{1}{2}$
4th. 918 $\frac{1}{2}$ — $\frac{1}{2}$ d.	1	18	$8\frac{1}{2}$
5th. 739 $\frac{1}{2}$ —1d.	3	1	$7\frac{1}{2}$
6th. 567 $\frac{1}{2}$ —1 $\frac{1}{2}$ d.	3	10	$11\frac{1}{2}$
7th. 685 $\frac{1}{2}$ —2d.	5	14	$8\frac{1}{2}$
8th. 475 $\frac{1}{2}$ —4d.	7	18	5
9th. 919 $\frac{1}{2}$ —6d.	22	16	9

CASE II.

When the price is pence, and no even part of a shilling:
Find the value of the given quantity at 1s. per yard;
divide the pence into aliquot parts, for divisors;
and the sum of the quotients, arising from them, will
be the answer.

EXAMPLES.

1. What will $487\frac{1}{2}$ yards come to, at 5d. per yard?

	£.	s.	d.	
3d. $\frac{1}{4}$	24	7	6	value of $487\frac{1}{2}$ yards, at 1s. per yd.
2d. $\frac{1}{6}$	6	1	$10\frac{1}{2}$	value of ditto, at 3d. per yard.
	4	1	8	value of ditto, at 2d. per yard.

Ans. £ 10 3 $1\frac{1}{2}$ value of ditto, at 5d. per yard.

Questions.

Answers.

Yds.	£.	s.	d.
2d. 568 $\frac{1}{2}$ at 7d.	16	11	$5\frac{1}{2}$
3d. 683 $\frac{1}{2}$ —8d.	22	15	0
4th. 912 $\frac{1}{2}$ —9d.	34	4	$4\frac{1}{2}$
5th. 649 $\frac{1}{2}$ —10d.	27	12	$0\frac{1}{2}$
6th. 745 $\frac{1}{2}$ —11d.	34	9	$7\frac{1}{2}$

CASE

CASE III.

When the price is pence or farthings, and an even part of a pound, cut off the right hand figure of the given quantity; and the cypher, in the aliquot part (if it has one) and divide by the remaining figure or figures: When you come to the remainder, double it; and divide as before: The answer will be pounds, shillings, &c. If there be no cypher in the divisor, then none should be cut off from the dividend.

EXAMPLES.

$$\begin{array}{r} d. \\ 1\frac{1}{2} | 1\frac{1}{2}.0 | 3795 \text{ lbs. at } \frac{1}{4}d. \\ 96 = 8 \times 12 \end{array} \quad \begin{array}{r} 379 | 5 \\ 8 \overline{) 31 \ 12 \ 6} \end{array}$$

Ans. £3 19 0 $\frac{1}{2}$

$$\begin{array}{r} d. \\ 1\frac{1}{2} | 1\frac{1}{2}.0 | 3795 \text{ yds. at } \frac{1}{2}d. \\ 48 = 6 \times 8 \end{array} \quad \begin{array}{r} 379 | 5 \\ 6 \overline{) 47 \ 8 \ 9} \end{array}$$

Ans. £7 18 1 $\frac{1}{2}$

$$\begin{array}{r} d. \\ 1\frac{1}{2} | 1\frac{1}{2}.0 | 3795 \text{ yds. at } \frac{3}{4}d. \\ 32 = 4 \times 8 \end{array} \quad \begin{array}{r} 379 | 5 \\ 4 \overline{) 47 \ 8 \ 9} \end{array}$$

Ans. £11 17 2 $\frac{1}{2}$

$$\begin{array}{r} d. \\ 1 | 1\frac{1}{2}.0 | 3795 \text{ yds. at } 1d. \\ 24 = 4 \times 6 \end{array} \quad \begin{array}{r} 379 | 5 \\ 4 \overline{) 63 \ 5} \end{array}$$

Ans. £15 16 3

$$\begin{array}{r} d. \\ 1\frac{1}{2} | 1\frac{1}{2}.0 | 3795 \text{ ds. at } 1\frac{1}{2}d. \\ 16 = 4 \times 4 \end{array} \quad \begin{array}{r} 379 | 5 \\ 4 \overline{) 94 \ 17 \ 6} \end{array}$$

Ans. £23 14 4 $\frac{1}{2}$

$$\begin{array}{r} d. \\ 2 | 1\frac{1}{2}.0 | 379 | 5 \text{ yds. at } 2d. \\ 12 = 3 \times 4 \end{array} \quad \begin{array}{r} 379 | 5 \\ 3 \overline{) 125 \ 6} \end{array}$$

Ans. £47 8 9

* In dividing by 12, there remains 7 before the stroke, which separates the 5; then 75 doubled, is 150; and 12 is found 12 $\frac{1}{2}$ times in 150; which makes 125. 6d.

d.
 $14 \frac{1}{2} \text{ } | 379 \frac{1}{2} \text{ yds. at } 4d. \text{ } d.$
£63 5 *Ans.*

$16 \frac{1}{2} \text{ } | 379 \frac{1}{2} \text{ yds. at } 6d.$
£94 17 6 *Ans.*

d.
 $18 \frac{1}{2} \text{ } | 379 \frac{1}{2} \text{ yds. at } 8d. \text{ } d.$
£126 10 *Ans.*

$10 \frac{1}{2} \text{ } | 3795 \text{ yds.}$
 $24 = 4 \times 6 \text{ } | 3795$

4) 632 100

£158 2 6 *Ans.*

C A S E IV.

When the price is between one and two shillings ;—
 Find the value of the quantity at 1s. per yard, &c.,
 which value, being divided by those even parts which
 the pence are of 1s. and the quotient or quotients,
 arising therefrom, added thereto ;—the sum will be
 the answer.

E X A M P L E S :

1. What will $758 \frac{1}{2}$ yards, at 1s 9 per yard, come to ?

£. s. d.
 $\left| \begin{array}{l} 6d. \text{ } | \frac{1}{2} \text{ } | 37 \text{ } 18 \text{ } 6 \text{ value at } 1s. \text{ per yard.} \\ 3d. \text{ } | \frac{1}{4} \text{ } | 18 \text{ } 19 \text{ } 3 \text{ value at } 6d. \text{ per yard.} \\ \quad \quad \quad 9 \text{ } 9 \text{ } 7 \frac{1}{2} \text{ value at } 3d. \text{ per yard.} \end{array} \right.$

Ans. £66 7 4½ val. of $758 \frac{1}{2}$ yds. at 1s 9 per yds.

Questions.

Answers.

Yds. £. s. d.
 2d. $987 \frac{1}{2}$ at $12 \frac{1}{2}d.$ — 51 8 7½
 3d. 793 — $12 \frac{1}{2}d.$ — 42 2 6½
 4th. $847 \frac{1}{2}$ — 15 1d. — 45 18 1½

5th.

Questions.

Answers.

Yds.

£. s. d.

5th. $680\frac{1}{4}$ at $1s. 1\frac{1}{2}d.$ — 88 12 76th. $591\frac{1}{4}$ — $1s. 2d.$ — 84 9 $9\frac{1}{4}$ 7th. $573\frac{1}{4}$ — $1s. 3d.$ — 85 16 $10\frac{1}{4}$

C A S E V.

When the price is any even number of shillings under 40 ;—Multiply the given quantity, by half the price ; and double the first figure of the product for shillings ; the rest of the product will be pounds.

N. B. If the price be 2s. you need only double the unit figure for shillings ; the other figures will be pounds.

E X A M P L E S.

1. What will 746 yards cost, at 2s. per yard ?

746

Ans. £ 74 12 value at 2s. per yard.

Note. The above is done, by saying, twice 6 (the unit figure) is 12 ; the other figures, viz. 74, are pounds.

2. What will $567\frac{1}{4}$ yards, at 2s. per yard, come to ?

Ans. £ 56 15/6.

N. B. Before I double the unit figure, viz. 7, I consider, that $\frac{1}{4}$ of a yard, at 2s. peryard, will amount to $1/6$; then, I double 7, which makes 14s. and $1/6$ added, makes $15/6$; the other figures are pounds.

Questions.

Answers.

Yds.

£. s. d.

3d. $129\frac{1}{4}$ at 4s. per yard, 25 18 0

4th. 697 — 6s. — 209 2 0

5th. 845 — 8s. — 338 0 0

6th. $917\frac{1}{4}$ — 10s. — 458 12 6

C A S E

PRACTICE.

CASE VI.

When the price wants an even part of 2s. — First, find the value of the whole quantity at 2s. per lb, yard, &c. then divide it by that even part which is wanting; and subtract this quotient from the value at 2s. the remainder will be the answer.

EXAMPLES.

1. What will $95\frac{1}{2}$ yards cost, at 23d. per yard?

£. s. d.
[2d. | $\frac{1}{16}$ | 9 11 0 value at 2s. per yard.
0 15 11 ditto at 2d. per yard.

Ans. £8 15 1 value at 1s. 10d. per yard.

Questions. Answers.

Yds.		£. s. d.
2d. 64	at 23d. per yard	6 12 8
3d. 128	— 22 $\frac{1}{2}$ d. —	12 0 0
4th. 246 $\frac{1}{2}$	— 21d. —	21 11 4 $\frac{1}{2}$
5th. 375 $\frac{1}{2}$	— 20d. —	32 5 5

CASE VII.

When the price is between 2s. and 3s. — First, find the value of the quantity at 2s. per yard, &c. which value being divided by those even parts, which the pence are of 2s. and those quotients added thereto, the sum will be the answer.

EXAMPLES.

1. What will 148 $\frac{1}{2}$ yards come to, at 2s. 7d. per yard?

[4d. | $\frac{1}{6}$ | 14 17 0 value at 2s. per yard.
[3d. | $\frac{1}{8}$ | 2 9 6 ditto at 4d. per yard.
1 17 1 $\frac{1}{2}$ ditto at 3d. per yard.

Ans. £19 3 7 $\frac{1}{2}$ value at 2s. 7d. per yard.

Questions.

PRACTICE.

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Questions.

Answers.

Yds.					
2d.	266½	at	2s. 1d.	per yard.	27 14 8½
3d.	844	—	2s. 1½d.	—	36 11 0
4th.	543½	—	2s. 2d.	—	58 17 7
5th.	655½	—	2s. 3d.	—	73 15 5½

CASE VIII.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20 ;—First, find the value of the quantity at the shillings per yard, &c. according to Case 5th ; then, suppose the quantity to stand as shillings per yard ; divide it, by that even part, which the pence are of 1s. and this quotient being added to the value before found, the sum will be the answer.

EXAMPLES.

1. What will 156½ yards come to, at 6s. 4d. per yard ?

Yds.	
156½	
d.	s. d.
14½	156 6

£46 19 0 value of 156½ yds. at 6s. per yd.
52s. 2d. = 2 12 2 value of ditto at 4d. per yd.

Ans. £49 11 2 value of ditto at 6¼ per yard.

Questions.

Answers.

Yds.					
2d.	17½	at	4 0½	per yard	3 10 8½
3d.	59½	—	6 0½	—	18 2 2½
4th.	68½	—	8 1	—	27 11 8½
5th.	96	—	10 1½	—	48 12 0

CASE IX.

When the price is any odd number of shillings under 40 ;—Find the value of the greatest even number contained

contained in the price, according to ease 5th, and add thereto the value of the quantity at 1s. per yard, &c. which sum will be the answer : Or, Multiply the quantity by the price, according to the 1st or 2d Case, in Simple Multiplication ; and divide the product by 20 ; the quotient will be the answer : Or, lastly, if the price be under 12s. find the value of the quantity at 1s. per yard, &c. and multiply it by the number of shillings in the price of 1 yard : The product will be the answer.

EXAMPLES.

1. What will 186 yards cost, at 3s. per yard ?
 £. s. Or thus :
 18 12 value at 2s. per yd. £. s.
 9 6 value at 1s. per yd. 9 6 val. at 1s. per yd.
 ——— 8
 £27 18 Ans. Product £27 18 Answer.

2. What will 647 yards cost, at 17s. per yard ?
 8

£517 12 value at 16s. per yard.
 32 7 value at 1s per yard.
 ———
 Ans. £549 19 value at 17s. per yard.

Questions.			Answers.		
Yds.	s.		£.	s.	d.
3d. 169½	at 5	per yard,	42	6	3
4th. 248½	— 7	- - -	87	1	3
5th. 139	— 9	- - -	62	11	0
6th. 782	— 25	- - -	977	10	0

C A S E X.

When the price is an even part of a pound :—Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide it by that part : the quotient will be the answer.

EXAMPLES.

PRACTICE.

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EXAMPLES.

1. What will $156\frac{1}{2}$ yards of cloth come to, at $3s. 4d.$ per yard?

$$\begin{array}{r} \text{£} \quad s. \quad d. \\ |3s. 4d.] \frac{1}{2} | 156 \quad 15 \quad 0 \text{ price at £1 per yard.} \end{array}$$

Ans. £26 2 6 price at $3s. 4d.$ per yard.

Questions.

	Yds.	s. d.	
2d.	$516\frac{1}{2}$	at 1 0	per yard.
3d.	624	— 1 3	_____
4th.	$719\frac{1}{2}$	— 1 4	_____
5th.	648	— 1 8	_____

Answers.

£.	s.	d.
26	16	9
89	0	0
47	19	4
54	0	0

CASE XI.

When the price wants an even part of a pound:—First find the value of the given quantity at £1 per yard, &c. then divide it by that even part which is wanting, and subtract this quotient therefrom: The remainder will be the answer.

EXAMPLES.

1. What will $167\frac{1}{2}$ yards cost, at $17s. 6d.$ per yard?

$$\begin{array}{r} s. \quad d. \quad \text{£.} \quad s. \quad d. \\ |2 \quad 6 | \frac{1}{2} | 167 \quad 10 \quad 0 \text{ value at £1 per yard.} \\ \quad \quad \quad 20 \quad 18 \quad 9 \text{ value at } 2s. 6d. \text{ per yard.} \end{array}$$

Ans. £146 11 3 value at $17s. 6d.$ per yard.

Questions.

	Yds.	s. d.	
2d.	$347\frac{1}{2}$	at 13 4	per yard.
3d.	$484\frac{1}{2}$	— 15 0	_____
4th.	614	— 16 0	_____
5th.	$912\frac{1}{2}$	— 17 6	_____

Answers.

£.	s.	d.
231	13	4
261	6	3
491	4	0
798	4	$4\frac{1}{2}$

CASE

C A S E XII.

When the price is shillings, pence and farthings, and not an even part of a pound :—Multiply the given quantity by the shillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their sum will be the answer, in shillings, &c. Or, you may let the given quantity stand as pounds, per yard, &c. then draw a line underneath, and take parts of parts therefrom ; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways ; by which means he will be able to discover the most concise method of performing such questions in business, as may fall under this case.

What will $248\frac{1}{2}$ yards, at 7s. 6d. per yard, come to?

$$\begin{array}{r} d. \quad s. \quad d. \\ [6] \frac{1}{2} | 248 \quad 6 \text{ value of } 248\frac{1}{2} \text{ yards, at } 1s. \text{ per yard.} \\ 7 \end{array}$$

1739 6 value of ditto, at 7s. per yard.

124 8 value of ditto, at 6d. per yard.

$$\underline{210)186[3 \quad 9}$$

Ans. £98 8 9 value of ditto, at 7s. 6d. per yard.

Or, thus :

$$\begin{array}{r} d. \quad \text{£.} \quad s. \quad d. \\ [6] \frac{1}{2} | 12 \quad 8 \quad 6 \text{ value of } 248\frac{1}{2} \text{ yards, at } 1s. \text{ per yard.} \\ \text{Multi. by} \quad 7 \end{array}$$

86 19 6 value of ditto, at 7s. per yard.

6 4 8 value of ditto, at 6d. per yard.

$$\underline{\text{Ans.} \quad \text{£}98 \quad 8 \quad 9}$$

By the latter part of this Case.

s. d.	£.	s. d.	
50 $\frac{1}{4}$	248	10 0	value of 248 $\frac{1}{2}$ yards, at £1 per yard.
26 $\frac{1}{2}$	62	2 6	value of ditto, at 5s. per yard.
	31	1 3	value of ditto, at 2s. 6d. per yard.

Ans. £93 3 9 value of ditto, at 7s. 6d. per yard.

Questions.

Answers.

	Yds.		s. d.		£. s. d.
2d.	68 $\frac{1}{2}$	at	4 6	per yard.	15 8 3
3d.	124	—	5 8	- - - -	35 2 8
4th.	146	—	14 9	- - - -	107 13 6
5th.	218 $\frac{1}{2}$	—	12 6	- - - -	135 11 3

C A S E XIII.

When the price of the yard, &c. is pounds, shillings and pence ;—First multiply the quantity by the pounds ; and if the shillings and pence be an even part of a pound, divide the given quantity by that part, and add the quotient to the product for the answer : But if they be not an even part of a pound, you must take parts of parts, and add them together as before.—Or, Reduce the pounds and shillings into shillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their sum will be the answer in shillings, &c.

N. B. The learner should work the following questions both ways.

E X A M P L E S.

1. What will 156 yards of broad cloth come to, at £3 6/8 per yard?

£ 156 0 0 value at one pound per yard.

3
468 0 0
52 0 0
—
520 0 0

Ans. £520 0 0

Questions.

Questions.				Answers.		
	Yds.	£. s. d.		£.	s.	d.
2d.	345 $\frac{1}{2}$	at 6 5 0	per yard.	2159	7	6
3d.	59 $\frac{1}{2}$	— 3 6 8	—	199	3	4
4th.	75	— 5 3 4	—	387	10	0
5th.	08	— 4 6 0	—	392	8	0

C A S E XIV.

When the quantity is any number less than 1000 and the price not more than 12d. per yard, &c.—Find the value of the whole quantity at 1d. per yard, which may be done by dividing it by 12, mentally, setting down the quotient only in pounds, or shillings, or both: Then multiply this sum by the pence in the price of one yard, and the product will be the answer.

1. What will 759 $\frac{1}{2}$ yards cost, at 7d. per yard?

£. s. d.

0 63 3 $\frac{1}{2}$ value at 1d. per yard.

Or, 3 3 3 $\frac{1}{2}$ value at 1d. per yard.

Mult. by 7

Ans. £ 42 3 0 $\frac{1}{2}$ val^{ue} of 759 $\frac{1}{2}$ yards, at 7d. per yd.

Questions.				Answers.		
	Yds.	d.		£.	s.	d.
2d.	975 $\frac{1}{2}$	at 2	per yard.	8	2	2
3d.	846	— 3 $\frac{1}{2}$	—	12	6	9
4th.	793 $\frac{1}{2}$	— 4 $\frac{1}{2}$	—	15	14	2 $\frac{1}{2}$
5th.	684	— 5 $\frac{1}{2}$	—	15	13	6

C A S E XV.

When the price is such a number of shillings and pence, as, when reduced into pence, may be produced by any two numbers in the Multiplication Table, and when the quantity does not exceed 1000;—First find the value of the whole at 1d. per yard, &c. according to the last Case; then, multiply this sum by the component parts of the pence in the price, and the last product will be the answer.

EXAMPLES.

EXAMPLES.

1. What will 439½ yards cost, at 6s. 9d. per yard?

$$\begin{array}{r}
 \text{£. s. d.} \\
 1 \ 16 \ 7\frac{1}{2} \text{ value at 1d. per yard.} \\
 9 \\
 \hline
 16 \ 9 \ 7\frac{1}{2} \\
 9 \\
 \hline
 \end{array}$$

Ans. £ 148 6 7½ value at 8½d. or 6s. 9d. per yard.

N. B. In 6s. 9d. there being 8½ pence, 1 multiply by 9 twice, because 9 times 9 is 8½.

Questions.

Answers.

	Yds.	s. d.		£. s. d.
2d.	984½	at 1 2	per yard.	57 8 7
3d.	849½	— 2 8	—	113 6 0
4th.	657	— 3 6	—	114 19 6
5th.	593	— 4 8	—	128 7 4

CASE XVI.

When the quantity is 240 — As many pence as there are in the price of 1 yard, &c. so many pounds will the quantity amount to.

N. B. One farthing per yard will come to 5s. at halfpenny per yard, to 10s. and at three farthings to 15s.

EXAMPLES.

1. What will 240 yards come to, at 2s. 7½d. per yard?

Ans. £ 31 10s.

N. B. The price is 2s 7½ per yard :—Now, as in 2s 7 there are 31 pence, so the quantity being 240, comes to 31 pounds ; then, according to the same rule, the halfpenny per yard comes to 10s. therefore, the answer to the question is £ 31 10s.

Questions.

Answers.

	Yds.	s. d.		£. s.
2d.	240	at 1 7½	per yard.	19 15
3d.	240	— 2 9	—	33 0
4th.	240	— 3 4	—	40 0
5th.	240	— 7 8½	—	92 10

CASE

C A S E XVII.

When the quantity is not less than 228, nor more than 252 ;—First find the value of 240 yards, &c. by the last Case ; then multiply the price of 1 yard by the number above or under 240, and add or subtract this product, to or from the value of 240 yards, as the question may require ; and the sum or remainder will be the answer.

E X A M P L E S.

1. What will 248 yards come to, at 16s. 5½d. per yard ?

	£.	s.	d.	
	197	10	0	value of 240 yards.
16/5½ multiplied by 8	=	6	11	8 value of 8 yards.

Ans. £204 1 8 value of 248 yards.

2. What will 229 yards cost, at 5s. 9½d. per yard ?

	69	15	0	value of 240 yards.
5/9½ multiplied by 11	=	3	3	11½ value of 11 yards.

Ans. £66 11 0½ value of 229 yards.

Questions.

Yds.	s.	d.		Answers.		
				£.	s.	d.
3d. 228	at	6	10½	per yard.	78	7 6
4th. 252	—	4	6	—	56	14 0

C A S E XVIII.

When the price of one hundred weight is of several denominations, and the quantity likewise ;—Multiply the price by the integers, (that is the whole numbers), and take parts for the rest from the price of an integer ; which, added together, will be the answer.

E X A M P L E S.

1. What will 9 Cwt. 3 qrs. 14 lb of sugar come to, at £4 17s. 4d. per Cwt. ?

qrs.

grs. lb.	£. s. d.	
2 0 $\frac{1}{2}$	4 17 4	price of 1 Cwt.
1 0 $\frac{1}{2}$	9	
0 14 $\frac{1}{2}$	—	
	43 16 0	price of 9 0 0
	2 8 8	price of 0 2 0
	1 4 4	price of 0 1 0
	0 12 2	price of 0 0 14
	—	
	Ans. £48	1 2 price of 9 8 14

Questions.	Answers.
Cwt. grs. lb.	£. s. d.
2d. 8 1 16 tobacco, at 5 17 9 per cwt.	49 8 2 $\frac{1}{2}$
3d. 7 3 19 ————	60 9 0 $\frac{1}{2}$
4th. 12 1 24 ————	49 2 7 $\frac{1}{2}$
5th. 16 2 17 ————	40 11 1

C A S E XIX.

When the price is at any of the rates in the second Practice Table of aliquot parts :—Multiply the given quantity by the numerator, and divide that product by the denominator ; if the price be pence, the quotient will be the answer in shillings ; if shillings, the answer will be pounds.

E X A M P L E S.

1. What will 379 yards, at 4 $\frac{1}{2}$ d. per yard, come to ?

$$\begin{array}{r} 379 \\ 3 \\ \hline 8)1137 \\ \hline 2)114\frac{1}{2} \end{array}$$

Ans. £7 2 1 $\frac{1}{2}$

2. What will 149 yards, at 6s. per yard, come to ?

$$\begin{array}{r} 149 \\ 3 \\ \hline 1)p44\frac{1}{2} \\ \hline \text{Ans. £44 } 14 \end{array}$$

Questions.

Questions.				Answers.			
	Yds.	s.	d.	£.	s.	d.	
3d.	127 at	0	7½	per yard.	3	19	4½
4th.	159 —	0	8	—	5	6	0
5th.	173 —	0	9	—	6	9	9
6th.	241 —	0	10	—	10	0	10
7th.	249 —	7	6	—	93	7	6
8th.	357 —	12	6	—	223	2	6
9th.	345 —	13	4	—	230	0	0

C A S E XX.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money :—Annex a cypher to the money, and divide it by half the price, and the quotient will be the quantity to be purchased.

E X A M P L E S.

1. How many yards of cloth, at 18s. per yard, may I have for £345 ?

Half the price = 9 | 3450 = Money with a cypher annexed.

383½ Yards, Answer.

Questions.				Answers.			
		s.		£.	Yds.		
2d.	How many yds. at 2 peryd. for 427 ?			427	—	4270	
3d.	—	4	—	312	—	1560	
4th.	—	6	—	917	—	3056½	
5th.	—	8	—	195	—	487½	

C A S E XXI.

To find the discount of any invoice, or bill of parcels, at any rate per cent. — Multiply the pounds in the invoice by the amount of the discount of one pound, at the rate per cent. and take parts for the shillings and pence ; then add them together, and the sum will be the discount required.

N. B. The discount for 1 pound at any rate per cent. is in the 3d Practice Table, page 184.

E X A M P L E S.

PRACTICE.

201

EXAMPLES.

1. What is the discount of an invoice, amounting to £65 13s. 4d. at $7\frac{1}{2}$ per cent. ?

OPERATION.

	£.	s.	d.
The discount of £65 at 5 per cent. is	3	5	0
The discount of £65 at $2\frac{1}{2}$ per cent. is	-	1	12 6
The discount of 10s. being half a pound, at $7\frac{1}{2}$ per cent. is	-	-	0 0 9
The discount of 3s. 4d. being $\frac{1}{4}$ of a pound at $7\frac{1}{2}$ per cent. is	-	-	0 0 3

The sum is, £4 18 6
(Answer.)

N. B. When the rate per cent. is any even part of £100, it may be performed, by dividing the amount by *that even part*.

2. What is the discount of an invoice amounting to £57 13s. 9d. at $12\frac{1}{2}$ per cent. ?

	£.	s.	d.
$12\frac{1}{2} \mid \frac{1}{5} \mid 57 \ 13 \ 9$			

£7 4 $2\frac{1}{2}$ Answer.

CASE XXII.

To find the value of goods sold by particular quantities, viz. I. By the Score. II. Round Timber. III. By 5 Score to the Hundred. IV. By 112 to the Hundred. V. By 6 Score to the Hundred. VI. By the great Gross. VII. By the 1000.

I. To find the value of goods sold by the Score.

The price of one is given, to find the price of one score.

If the given price be shillings and pence, or only pence—Divide the given price, in pence, by 12 : The quotient will be the answer in pounds, and the remainder will be so many times $1/8$.

EXAMPLES.

EXAMPLES.

1. At 9d. each; What is that per score? 2. At 4/9 each; what is that per score?

12)9d. (.75 = £0 15 0 *Ans.*

4/9

Or, by inverting the question. 12

1 Score = 20 = 1/8

9

12)57d.

15 for

£4 15 *Answer.*

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1/8 as there are pence. If farthings are given; for 1/4d. reckon 5d. for 1/4d. 10d. and for 1/2d. 1/3.

TABLE of aliquot parts. 20 the Integer.

2 is $\frac{1}{10}$	6 is $\frac{3}{10}$	12 is $\frac{6}{10}$	16 is $\frac{8}{10}$
4 — $\frac{2}{10}$	8 — $\frac{4}{10}$	14 — $\frac{7}{10}$	18 — $\frac{9}{10}$
5 — $\frac{1}{2}$	10 — $\frac{1}{1}$	15 — $\frac{3}{2}$	

3. What cost 17, at 19/10 per score?

	s.	d.
10	19	10
5	9	11
2	4	11 1/2
—	1	11 3/4
17	16	10 3/4

II. Round Timber.

Forty feet make a load, or ton, of round timber.

If the given price of a foot be shillings;—Multiply the given price by 2, and the product will be the answer in pounds.

4. What cost a ton, at 3s. per foot?

5. What cost a ton, at 9s. per foot?

3s. $\times 2 =$ £6 *Answer.*

9s. $\times 2 =$ £18 *Ans.*

If the given price of a foot be pence only, or shillings and pence—Divide the given price, in pence,

pence, by 6; the quotient will be the answer in pounds, and the remainder will be so many times

$\frac{3}{4}$.

6. What cost 40 feet, at $17d.$ per foot?

$$\begin{array}{r} 6 \overline{) 17} \\ \underline{12} \\ 5 \end{array}$$

$\pounds 2 \ 16 \ 8$ Answer.

7. At $1/9$ per foot, What cost a ton?

$$\begin{array}{r} 6 \overline{) 21} \\ \underline{12} \\ 9 \end{array}$$

$\pounds 3 \ 10$ Answer.

If the given price of a foot be farthings only, or pence and farthings—Divide the given price, in farthings, by 6, then, divide that quotient by 4, and this last quotient will be the answer.

8. At $\frac{1}{4}d.$ per foot, What cost a ton?

$$\begin{array}{r} 6 \overline{) 3} \\ \underline{0} \\ 3 \end{array}$$

$$\begin{array}{r} 4 \overline{) 10} \\ \underline{8} \\ 2 \end{array}$$

$\pounds - \ 2 \ 6$ Answer.

9. At $13\frac{1}{4}d.$ per foot, What cost a ton?

$$\begin{array}{r} 12\frac{1}{4} \\ 4 \end{array}$$

$$\begin{array}{r} 6 \overline{) 53} \\ \underline{48} \\ 5 \end{array}$$

$$\begin{array}{r} 4 \overline{) 8 \ 16 \ 8} \\ \underline{4} \quad \underline{12} \quad \underline{8} \\ 0 \quad 0 \quad 0 \end{array}$$

$\pounds 2 \ 4 \ 2$ Answer.

Or, Suppose every shilling in the price to be so many times $\pounds 2$: Every penny to be so many times $\frac{3}{4}$: And every farthing to be $10d.$

10. What cost 40 feet, at $\frac{1}{4}d.$ per foot?

11. What cost 40 at $15\frac{1}{2}d.$ per foot?

$$\frac{1}{4}d. \times 10 = \pounds 0 \ 2 \ 6 \text{ Answer.}$$

$$1 \ 0 \times 2 = \pounds 2 \ 0 \ 0$$

$$3 \ 4 \times 3 = 0 \ 10 \ 0$$

$$\frac{1}{2} \times 10 = 0 \ 1 \ 8$$

$\pounds 2 \ 11 \ 8$ Ans.

III. To find the value of goods sold by 5 Score to the Hundred.

1. If the given price be pounds and shillings, or shillings only—Multiply the given price, in shillings, by 5; and the quotient will be the answer, in pounds.

12. At

12. At 19s. per yard,
What cost 100 yards?

$$\begin{array}{r} 19s. \\ 5 \\ \hline \end{array}$$

£95 *Ans.*

13. At £4 13s. per
cwt. What cost 100
cwt. or 5 tons?

$$\begin{array}{r} 4 \quad 13 \\ 20 \\ \hline 93 \\ 5 \\ \hline \end{array}$$

£465 *Ans.*

2. If the given price of 1, be pence only, or shillings and pence—Multiply the given price, in pence, by 5; then divide that product by 12; the quotient will be pounds, and the remainder so many times 1s. 8d.

14. If 1 yard cost 9d.; What cost 100 yards?

$$\begin{array}{r} 9 \\ 5 \\ \hline 12 \overline{)45} \\ \hline \end{array}$$

£3 15 *Ans.*

3. If the given price of 1, be shillings and pence—Multiply the price by 5, and the product, under the place of shillings, will be the answer in pounds; and the product under the place of pence, will be so many times 1s. 8d.

15. At 2s 5 per bushel;
What cost 100 bushels?

$$\begin{array}{r} s. \quad d. \\ 2 \quad 5 \\ 5 \\ \hline 12 \quad 1 \\ \hline \end{array}$$

Ans. £12 1 8

16. At 25s 3 per ton;
What cost 100 tons?

$$\begin{array}{r} s. \quad d. \\ 25 \quad 3 \\ 5 \\ \hline 126 \quad 3 \frac{1}{8} \times 3 = 5 \frac{1}{8} \\ \hline \end{array}$$

£126 5 *Ans.*

IV. 70

4. To find the price of one, at so much per hundred of 5 score.—Multiply the given price by 12; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only; Divide the given price by 5, and the quotient will be the answer in shillings.

17. If 100 yards cost £65; What cost 1 yard?

$$\begin{array}{r} 5 \overline{)65} \\ \underline{50} \\ 15 \end{array}$$

13s. Ans.

$$\begin{array}{r} £11 \ 7 \ 9 \\ 12 \end{array}$$

$$\begin{array}{r} 5 \overline{)126 \ 18 \ 0} \\ \underline{100} \\ 26 \end{array}$$

$$\begin{array}{r} 12 \overline{)27 \ 6 \ 7} \\ \underline{24} \\ 3 \end{array}$$

18. If 100 yards cost £11 7s. 9d. What cost 1 yard?

2s. 3½d. Ans.

In dividing 27 by 12 (in the 18th question) the quotient is 2s. and the remainder 3d.—the 6 is $\frac{3}{10}$ of a penny = two farthings, and the 7 is of no account.

TABLE of Aliquot Parts. 100 the Integer.

5 is $\frac{1}{20}$	25 is $\frac{1}{4}$	50 is $\frac{1}{2}$	75 is $\frac{3}{4}$
10 — $\frac{1}{10}$	30 — $\frac{3}{10}$	60 — $\frac{3}{5}$	80 — $\frac{4}{5}$
20 — $\frac{1}{5}$	40 — $\frac{2}{5}$	70 — $\frac{7}{10}$	90 — $\frac{9}{10}$

19. At £3 7/6 per 100; What will 23 cost?

		£. s. d.	
20	$\frac{1}{5}$	3 7 6	
		0 13 6	
2	$\frac{1}{10}$	0 1 4	} Add.
1	$\frac{1}{20}$	0 0 9	

$$23 = £0 \ 15 \ 6 \text{ Ans.}$$

IV. To find the value of goods sold by 112½ the cwt.

The price of 1lb is given to find the value of a cwt.

RULE—For a farthing, account $\frac{2}{4}$ per cwt. For a half penny, $\frac{4}{8}$. For three farthings, $\frac{7}{8}$, and for every penny, $\frac{9}{4}$ per cwt.

20. What cost 1 cwt.

at $3\frac{1}{4}d.$ per lb?

At $1d.$ per lb. s. d.

1 cwt. costs 094
3

At $3d.$ £1 8 0 }
At $\frac{1}{4}d.$ £0 4 8 } Add.

£1 12 8 Ans.

21. At $8\frac{1}{4}d.$ per lb

What cost 1 cwt.?

At $1d.$ per lb, s. d.

1 cwt. costs - 9 4
8

At $8d.$ £8 14 8 }
At $\frac{1}{4}d.$ £0 7 0 } Add.

£4 1 8 Ans.

V. To find the value of goods sold by 6 score to the hundred.

The price of 1 is given, to find the price of one hundred.

RULE.—Suppose every penny in the price to be so many pounds, and for the farthings, such a part of a pound, as they are of a penny; then, half of that sum will be the answer.

22. At $4\frac{1}{4}d.$ per yard,

What cost 120 yards?

£. s.
2)4 10

£2 5 Ans.

23. At 16s. $9\frac{1}{4}d.$ per yard,

What cost 120 yards?

16s. $9\frac{1}{4}d.$
12

201

£. s.
2)201 5

£100 12 5 Ans.

To find the price of one, at so much per hundred of 6 score.

RULE.—Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part

part of a penny, as they are of a pound, and you will have the answer.

24. If 120 yards cost £3 12s. ; What cost 1 yard ? £. s. d.

3 12

2

—

7 4

Ans. 7 1/2 d.

25. If 120 yards cost £5 18s. 6 ; What cost 1 ? £. s. d.

5 18 6

2

—

11 17 0

Ans. 11 1/2 d. + 2/3 of a farth.

TABLE of Aliquot Parts. 120 the Integer.

Also,

6 is 1/20	24 is 1/5	36 is 1/10	72 is 1/20	96 is 1/30
10 — 1/12	30 — 1/4	45 — 2/9	75 — 2/30	100 — 1/60
12 — 1/10	40 — 1/3	48 — 2/5	80 — 3/40	105 — 1/70
15 — 1/8	60 — 1/2	50 — 1/3	84 — 7/10	108 — 1/90
20 — 1/6		70 — 1/21	90 — 1/3	110 — 1/110

26. At £3 17s. 6d. per hundred ; What cost 14 ?

	£. s. d.
12	3 17 6
2	0 7 9
14	0 1 3 1/2

Add

14 = £0 9 0 1/2 Ans.

27. At £1 19s. 3d. per hundred ; What cost 75 ?

£. s. d.

80	16 8
5	9 6 1/2
75	14 6 1/2

Ans.

VI. To find the value of goods sold by the Great Grofs.

Note, 12 make 1 dozen, 12 dozen 1 small grofs, 12 small grofs 1 great grofs.

The price of 1 dozen being given, in pence, to find the price of a great grofs.

RULE.—Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c.

For proof, do the contrary.

N. B. If the price of 1 be given, the price of 1 small grofs is found after the same manner.

28. At 18d. per dozen; What cost 1 great grofs?

18d.

3

54

£10 15 Ans.

29. At 4s. 3d. per dozen; What cost 1 great grofs?

4s. 3d.

12

51d.

3

513

Ans. £30 12

Or,

s. d.

4 3

12

513

12

£30 12 0

TABLE of Aliquot Parts. 144 the Integer.

Also,

12 is $\frac{1}{12}$	36 is $\frac{1}{4}$	32 is $\frac{2}{5}$	84 is $\frac{2}{3}$	128 is $\frac{2}{3}$
24 is $\frac{1}{6}$	48 is $\frac{1}{3}$	60 is $\frac{2}{3}$	96 is $\frac{2}{3}$	132 is $\frac{2}{3}$
18 is $\frac{2}{9}$	72 is $\frac{2}{3}$	64 is $\frac{4}{5}$	108 is $\frac{4}{5}$	
24 is $\frac{1}{6}$		80 is $\frac{4}{5}$	120 is $\frac{4}{5}$	

30. What

30. What cost 117 dozen, at £9 13s. 7d. per great gross?

		<i>Doz.</i>			
108	$\frac{1}{4}$	29	0	9	
9	$\frac{1}{12}$	7	5	2	} <i>Add.</i>
		0	12	1	
117	=	£7		17	3 $\frac{1}{4}$ <i>Ans.</i>

31. At £3 16s. 8d. per great gross; What cost 7 great gross, and 96 dozen?

		<i>£. s. d.</i>
		3 16 8
		2
		3)7 13 4
<i>Doz.</i>	96	= 2 11 1 $\frac{1}{4}$
Top line $\times 7$		= 26 16 8
		£29 7 9 $\frac{1}{4}$ <i>Ans.</i>

VII. To find the value of goods sold by the Thousand.

The price of 1 is given, to find the price of 1000.

RULE.—Multiply the given price, in pence, by 50; then, divide the product by 12, and the quotient will be the answer in pounds, &c.

32. At 6d. each; Or, as 1000s. are £50; What cost 1000? take parts, for the pence, out of 50.

6	
50	
12)300	
£25	<i>Ans.</i>

To

To find the price of *one*, at so much per thousand.

RULE.—Multiply the price by 12; divide the product by 50; then take the pounds for so many pence, and the shillings, for such a part of a penny as they are of a pound, which will be the answer.

33. At £354 3s. 4d. per 1000; What cost 1?

£ s. d.	100	12	£ s. d.
354 3 4	100	12	354 3 4
12	10	12	85 8 4
10	10	12	8 10 10
50	10	12	Ans. 0 7 1

Ans. 7s. 1d.

TABLE of Aliquot Parts. — 1000 the Integer.

50 is $\frac{1}{20}$	200 is $\frac{1}{5}$	300 is $\frac{3}{10}$	700 is $\frac{7}{10}$
100 — $\frac{1}{10}$	250 — $\frac{1}{4}$	375 — $\frac{3}{8}$	750 — $\frac{3}{4}$
125 — $\frac{1}{8}$	500 — $\frac{1}{2}$	400 — $\frac{2}{5}$	800 — $\frac{4}{5}$
		600 — $\frac{3}{5}$	875 — $\frac{7}{8}$
		625 — $\frac{5}{8}$	900 — $\frac{9}{10}$

34. At £1 17s. 9d. per 1000; What cost 115?

35. At £2 1s. 8d. per 1000; What cost 875?

£ s. d.	100	12	£ s. d.
1 17 9	100	12	1 17 9
12	10	12	0 3 9 1
10	10	12	0 0 4 1
5	10	12	0 0 2 1
115	10	12	Ans. 0 4 4

£ s. d.	1000	12	£ s. d.
2 1 8	1000	12	2 1 8
12	10	12	0 14 8
10	10	12	0 16 5 1
875	10	12	Ans. 1 16 5 1

BILL

TARE AND TRET.

BILL of PARADES.

Newburyport, January 1st, 1793.

Mr. Timothy Huckster

Bought of Samuel Merchant,

25½ lb Bohea tea, at 3s. 6d. per lb.

48 lb cheese, at 9d. per lb.

15 pair of worsted hose, at 5s. 8d. per pair.

4 dozen women's gloves, at 36s. 6d. per dozen.

19 dozen knives and forks, at 5s. 9d. per dozen.

9 grindstones, at 15s. 9d. per stone.

½ cwt. brown sugar, at 5s. per cwt.

21 lb loaf sugar, at 13. ½d. per lb.

£34 8 8½

Received payment in full,

Samuel Merchant.

TARE and TRET.

Tare and Tret are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4 lb in every 104 lb for waste, dust, &c.

Cloff is an allowance of 2 lb upon every 3 cwt.

Gross weight is the whole weight of any sort of goods, together with the box, barrel or bag, &c. which contains them.

Nettle is when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

CASE

TARE AND TRET.

CASE I.

When the tare is at so much per box, barrel or bag, &c.—Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.

EXAMPLE 1.

1. In 6 hogheads of sugar, each weighing 9 cwt. 2 qrs. 11 lb gross, tare 25 lb per hoghead: How much neat?

Cwt.	qr.	lb	Cwt.	qr.	lb
25	×	6	=	1	1
				10	9
					2
					10
					gross wt. of 1 hhd.
					6

48 8 18

57 2 4 gross.
1 1 10 tare.

Ans. 56 0 22 neat.

2. In 5 bags of cotton, marked with the gross weight as follows, tare 23 lb per bag: What neat weight?

Cwt.	qrs.	lb
A	=	7 1 19
B	=	3 8 27
C	=	5 1 12
D	=	6 0 15
E	=	8 1 0

Cwt. qrs. lb
Ans. 30 0 14 neat.

3. What is the neat weight of 15 hogheads of tobacco, each 7 cwt. 1 qr. 13 lb, tare 100 lb per hhd?

Ans. 97 cwt. 0 qr. 11 lb.

CASE

This, as well as every other Case in this rule, is only an application of the rules of Proportion and Practice.

END

C A S E II.

When the tare is at so much per cwt.—Divide the gross weight by the aliquot parts of a cwt. Subtract the quotient from the gross, and the remainder will be the neat weight.

E X A M P L E S.

1. In 129 cwt. 3 qrs. 16 lb gross; tare 14 lb per cwt. What neat weight?

lb	Cwt.	qrs.	lb
14 $\frac{1}{8}$	129	3	16 gross.
	16	0	26 $\frac{1}{2}$ tare.

Ans. 113 2 17 $\frac{1}{2}$ neat.

2. In 97 cwt. 1 qr. 7 lb gross; tare, 20 lb per cwt. What neat weight?

lb		Cwt. qrs. lb	
16	$\frac{1}{4}$	97	1 7 gross.
4	$\frac{1}{4}$	13	3 17 } Add.
		3	1 25

Subtract 17 1 14 tare.

Ans. 79 3 21 neat.

3. What is the neat weight of 9 barrels of potash, each weighing 305 lb gross; tare 12 lb per cwt.?

Ans. 2450 lb 14 oz. 3 drs.

C A S E III.

When the tret is allowed with tare;—Divide the futtle weight by 46, and the quotient will be the tret, which subtract from the futtle, and the remainder will be the neat.

E X A M P L E S.

TABLE OF POWERS.

Roots,	1	2	3	4	5	6	7	8	9
Squares,	1	4	9	16	25	36	49	64	81
Cubes,	1	8	27	64	125	216	343	512	729
Biquadrates,	1	16	81	256	625	1296	2401	4096	6561
Surfolids,	1	32	243	1024	3125	7776	16807	32768	59049
Square Cubes,	1	64	729	4096	15625	46656	137649	262144	331441
Second Surfolids,	1	128	2187	16384	78125	179936	833543	2097152	4782969
Biquadrates Squared,	1	256	6561	65536	390625	1679816	5764801	16177216	43046721
Cubes Cubed,	1	512	29683	262144	1953125	10977696	40353607	134217728	387430489
Surfolids Squared,	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	4386784401
Third Surfolids,	1	1024	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square Cubes Squared,	1	4096	531441	16777216	24220625	2176782336	13841287201	68719476736	282429536489

EVOLUTION.

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EXAMPLES.

1. What is the 5th power of 9?

$$\begin{array}{r} 9 \\ 9 \\ \hline \end{array}$$

$$81 = 2^{\text{d}} \text{ power.}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$729 = 3^{\text{d}} \text{ power.}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$6561 = 4^{\text{th}} \text{ power.}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$59049 = 5^{\text{th}} \text{ power,}$$

$$\text{or } \text{Ans.} = 9^5.$$

2. What is the 5th power of $\frac{2}{3}$? *Ans.* $\frac{32}{243}$.

3. What is the 4th power of .045?

$$\text{Ans. } .00004109625.$$

Here, we see, that in raising a fraction to a higher power, we decrease its value.

EVOLUTION,

Or the EXTRACTION of ROOTS.

The Root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th root, &c. accordingly, as it is, when raised to the 2d, 3d, 4th, &c. power, equal to that power. Thus, 4 is the square root of 16; because $4 \times 4 = 16$, and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$; and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which precise roots can never be determined. But by the help of decimals, we can approximate towards the root, to any assigned degree of exactness.

T

The

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The roots, which approximate; are called *surd roots*, and those which are perfectly accurate, are called *rational roots*.

Roots are sometimes denoted by writing the character $\sqrt{}$ before the power, with the index of the root over it; thus the 3d root of 36 is expressed $\sqrt[3]{36}$, and the 2d root of 36 is $\sqrt{36}$, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers, with the sign $+$ or $-$ between them, a line is drawn from the top of the sign over all the parts of it; thus, the 3d root of $47 + 22$ is $\sqrt[3]{47+22}$, and the 2d root of $59 - 17$ is $\sqrt{59-17}$, &c.

Sometimes roots are designed like powers, with fractional indices; thus, the square root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and the 4th root of $37 - 20$ is $37 - 20^{\frac{1}{4}}$, &c.

The EXTRACTION of the SQUARE ROOT.

RULE 1.—Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek

EXTRACTION of the SQUARE ROOT. 219

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: Multiply the divisor, with the figure last annexed, by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figure already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed; and continue the operation, in the same manner, till you have brought down all the periods.

Note 1. If, when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

Note 2. If there be decimals in the given number, it must be pointed both ways from the place of units: If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers.

E X A M P L E S.

1. Required the square root of 30138696025?

30138696025 (173605 the root.

1st Divisor = 27)201

189

2d Divisor = 343)1238

1029

3d Divisor = 3466)20969

20796

4th Divisor = 347205)1736025

1736025

2. Required

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2. Required the square root of 575,5 ?

575,50(23,984 root,

4

43)175

129

469)4650

4221

4788)42900

38304

4596 remainder.

3. What is the square root of 10342656 ?

Ans. 3216.

4. What is the square root of 964,5192860241 ?

Ans. 31,05671.

5. What is the square root of ,0000316969 ?

Ans. ,00563.

Note. When more than half the root is found, the remaining figures of it may be found by division, making use of the last divisor, and bringing down so many of the next figures of the resolvend, as there were periods to come down, when you began the division.

RULES for the SQUARE ROOT of VULGAR FRACTIONS and MIXED NUMBERS.

After reducing the fraction to its lowest terms, for this and all other roots ; then,

1. Extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator, which is the best method, provided the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together ; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required :—

Or,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixed

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4. Mixed numbers may either be reduced to improper fractions, and extracted by the first or second rule; or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

E x a m p l e s.

1. What is the square root of $\frac{144}{16}$?

By Rule 1.

$$\frac{144}{16} = \frac{12}{4} \quad 16 \text{ (4 root of the numerator)}$$

$$1681 \quad 41 \text{ (4 root of the denominator)}$$

81)81 Therefore $\frac{12}{41}$ = the root of the given fraction.

By Rule 2.

$$16 \times 1681 = 26896 \text{ and } \sqrt{26896} = 164. \text{ Then,}$$

$$\frac{164}{16} = \frac{41}{4} = \frac{12}{41} = .09756 +$$

By Rule 3.

$$1681)16,0095181439+. \text{ And } \sqrt{16,0095181439} = .09756+.$$

2. What is the square root of $\frac{3721}{1600}$? *Ans.* $\frac{61}{40}$.

3. What is the square root of $42\frac{1}{4}$? *Ans.* $6\frac{1}{2}$.

Note. In extracting the square or cube root of any surd number, there is always a remainder or fraction left, when the root is found: To find the value of which, the common method is, to annex pairs of cyphers to the resolvend for the square, and ternaries of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube. Now this trouble may be saved by the following method.

In the square, the quotient is always doubled for a new divisor: Therefore, when the work is completed, the root doubled is the true divisor, or denominator* to its own fraction; as, if the root be

* These denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use.

12, the denominator will be 24 ; to be placed under the remainder ; which vulgar fraction, or its equivalent decimal, must be annexed to the quotient, or root, to complete it.

If to the remainder either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION and Use of the SQUARE ROOT.

PROBLEM I. *To find a mean proportional between two numbers.*

RULE.—Multiply the given numbers together, and extract the square root of the product ; which root will be the mean proportional sought.

EXAMPLE. What is the mean proportional between 24 and 96 ?

$$\sqrt{96 \times 24} = 48 \text{ Ans.}$$

PROBLEM II. *To find the side of a Square equal in Area to any given Superficies whatever.*

RULE.—Find the Area, and the square root is the side of the square sought.

1. If the area of a circle be 184.125, What is the side of a square equal in area thereto ?

$$\sqrt{184.125} = 13.569 + \text{Ans.}$$

2. If the area of a triangle be 160 ; What is the side of a square equal in area thereto ?

$$\sqrt{160} = 12.649 + \text{Ans.}$$

PROB.

PROB. III. A certain General has an army of 5625 men; Pray how many must he place in rank and file, to form them into a square?

$$\sqrt{5625} = 75 \text{ Ans.}$$

PROB. IV. Let 10952 men be so formed, as that the number in rank may be double the file.

$$\sqrt{\frac{10952}{2}} = 74 \text{ in file, and } 74 \times 2 = 148 \text{ in rank.}$$

PROB. V. If it be required to place 2016 men so as that there may be 56 in rank and 36 in file, and to stand 4 feet distance in rank, and as much in file. How much ground do they stand on?

To answer this, or any of the kind, use the following proportion:—As unity : to the distance :: so is the number in rank less by one : to a fourth number;—next, do the same by the file, and multiply the two numbers together, found by the above proportion, and the product will be the answer.†

As 1 : 4 :: 56 — 1 : 220. And as 1 : 4 :: 36 — 1 : 140. Then, 220 X 140 = 30800 square feet, the Answer.

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards; How many trees must it be in length, and how many in breadth; and how many square yards of ground do they stand on?

• If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file; extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

† The above rule will be found useful in planting trees, having the distance of ground between each given.

To resolve any question of this nature; say, as the ratio in length : is to the ratio in breadth :: so is the number of trees : to a fourth number; whose square root is the number in breadth;— And as the ratio in breadth : is to the ratio in length :: so is the number of trees : to a fourth, whose root is the number in length.

As 2 : 2 :: 600 : 400. And $\sqrt{400} = 20 =$ number in breadth.

As 2 : 3 :: 600 : 900. And $\sqrt{900} = 30 =$ number in length.

As 1 : 7 :: 30—1 : 203. And as 1 : 7 :: 20—1 : to 133. And $203 \times 133 = 26999$ square yards, the *Answer*.

PROB. VII. Admit a leaden pipe $\frac{1}{2}$ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe, which will fill the same cistern in 1 hour.

RULE.—As the given time is to the square of the given diameter, so is the required time, to the square of the required diameter. $3 = .75$; and $.75 \times .75 = .5625$; Then, As $3h : .5625 :: 1h : 1.6875$ inversely, and $\sqrt{1.6875} = 1.3$ inch nearly, *Answer*.

PROB. VIII. If a pipe, whose diameter is 1.5 inch, fill a cistern in 5 hours; In what time will a pipe, whose diameter is 3.5 inches fill the same?

$1.5 \times 1.5 = 2.25$; and $3.5 \times 3.5 = 12.25$; Then, As $2.25 : 5 :: 12.25 : .94$ hour, inversely, $= 54$ min. 36 sec. *Answer*.

PROB. IX. If a pipe, 6 inches bore, will be 4 hours in running off a certain quantity of water; In what time will 3 pipes, each 4 inches bore, be in discharging double the quantity?

$6 \times 6 = 36$. $4 \times 4 = 16$, and $16 \times 3 = 48$. Then, as $36 : 4h :: 48 : 3h$ inversely, and as 1w. : 3h. :: 2w. : 6h. *Answer*.

PROB. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE.—Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter ;—But if the required circle be less ; divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle, whose diameter is 4 inches ; I demand the diameter of a circle 3 times as large ?

$4 \times 4 = 16$; and $16 \times 3 = 48$; and $\sqrt{48} = 6,928 +$ inches, *Answer*.

PROB. XI. To find the diameter of a circle, equal in area to an ellipsis (or oval) whose transverse and conjugate diameters are given.*

RULE.—Multiply the two diameters of the ellipsis together ; and the Square Root of that product will be the diameter of a circle equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the conjugate 36 ; What is the diameter of an equal circle ?

$48 \times 36 = 1728$, and $\sqrt{1728} = 41,569 +$ the *Answer*.

PROB. XII. Two ships sail from the same port ; one goes due north 45 leagues, and the other due west 76 leagues : How far are they asunder ?

$45 \times 45 = 2025$. $76 \times 76 = 5776$. Then, $5776 \times 2025 = 7801$, and, $\sqrt{7801} = 88,32$ leagues, the *Answer*.

EXTRACTION

* The transverse and conjugate are the longest and shortest diameters of an ellipsis ; they pass through the centre, and cross each other at right angles.

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EXTRACTION of the CUBE ROOT.

A Cube, is any number multiplied by its square. To extract the cube root, is to find a number which being multiplied into its square, shall produce the given number.

FIRST METHOD.

RULE 1.—Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend*.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor*.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. Multiply the triple square by the last quotient figure, and write the product under this dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the *subtrahend*.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

Note. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root.

EXAMPLES.

EXTRACTION of the CUBE ROOT.

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EXAMPLES.

1. Required the cube root of 436086824287?

$$\begin{array}{r} 436086824287 \text{ the root.} \\ 343 \end{array}$$

1st Divisor = 1491093036 = 1st Dividend.

$$\begin{array}{r} 73500 \\ 8250 \\ 125 \end{array}$$

78875 = 1st Subtrahend.

2d Divisor = 168775014161824 = 2d Dividend.

$$\begin{array}{r} 13500000 \\ 144000 \\ 612 \end{array}$$

13644512 = 2d Subtrahend.

3d Divisor = 172391940517312287 = 3d Dividend.

$$\begin{array}{r} 517107600 \\ 204660 \\ 27 \end{array}$$

517312287 = 3d Subtrahend.

The Method of Operation.

$$7 \times 7 \times 300 = 24700 = 1st \text{ Triple square.}$$

$$7 \times 30 = 210 = 1st \text{ Triple quotient.}$$

$$34910 = 1st \text{ Divisor.}$$

$$14700 \times 5 = 73500$$

$$5 \times 5 \times 210 = 5250$$

$$5 \times 5 \times 5 = 125$$

78875 = 1st Subtrahend.

Carried over.

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Brought over.

$$75 \times 75 \times 300 = 1687500 = 2d \text{ Triple square.}$$

$$75 \times 30 = 2250 = 2d \text{ Triple quotient.}$$

$$1689750 = 2d \text{ Divisor.}$$

$$1687500 \times 8 = 13500000$$

$$2250 \times 8 \times 8 = 144000$$

$$8 \times 8 \times 8 = 512$$

$$13644512 = 2d \text{ Subtrahend.}$$

$$758 \times 758 \times 300 = 172369200 = 3d \text{ Triple square.}$$

$$758 \times 30 = 22740 = 3d \text{ Triple quotient.}$$

$$172391940 = 3d \text{ Divisor.}$$

$$172369200 \times 3 = 517107600$$

$$22740 \times 3 \times 3 = 204660$$

$$3 \times 3 \times 3 = 27$$

$$517312287 = 3d \text{ Subtrahend}$$

2. What is the cube root of 34965783 ?

Ans. 327.

3. What is the cube root of 84,604519 ?

Ans. 4,39.

4. What is the cube root of ,008649 ?

Ans. ,2052+.

5. What is the cube root of $\frac{125}{343}$?

Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finished.

In the extraction of the cube root, the quotient is said to be squared and tripled for a new divisor: But is not really so, till the triple number of the quotient be added to it; therefore, when the operation is finished, it is but squaring the quotient, or root, then multiplying it by 3, and to that number adding the triple number of the root, when it will become

EXTRACTION of the CUBE ROOT. 219

become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient, to complete the root.

Suppose the root to be 12, when squared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.

SECOND METHOD.

RULE 1.—Having pointed the given number into periods of three figures each, find the greatest cube in the left hand period, subtracting it therefrom, and placing its root in the quotient; to the remainder bring down the next period, and call it the *dividend*.

2. Under this dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; and under the said triple square, write the triple root, removed one place to the right hand, and call the sum of these the *divisor*.

3. Seek how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, setting down the unit's place of this line, under the place of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient figure, so as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the last quotient figure, and call their sum the *subtrahend*.

5. Subtract the subtrahend from the dividend, and to the remainder bring down the next period.

for a new dividend, with which proceed as before, and so on, till the whole be finished.

EXAMPLE.

Required the cube root of 16194277 P.

16194277 (253 = Root.

8

8194 = First dividend.

12

= Triple square of 2.

6

= Triple of 2.

126

= First divisor.

60

= Triple square of 2 multiplied by 3.

150

= Triple of 2 multiplied by the square of 3.

125

= Cube of 5.

7625

= First subtrahend.

569277 = Second dividend.

1875

= Triple square of 25.

75

= Triple of 25.

18825

= Second divisor.

5625

= Triple square of 25 multiplied by 3.

675

= Triple of 25 multiplied by square of 3.

27

= Cube of 3.

569277

= Second subtrahend.

FIRST METHOD by APPROXIMATION.

RULE 1.—Find by trial, a cube near to the given number, and call it the *supposed* cube.

2. Then,

EXTRACTION of the CUBE ROOT.

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2. Then, as twice the *supposed* cube, added to the given number, is to twice the given number, added to the *supposed* cube, so is the root of the *supposed* cube, to the true root, or an approximation to it.

3. By taking the cube of the root, thus found, for the *supposed* cube, and repeating the operation, the root will be had to a greater degree of exactness.

EXAMPLE.

It is required to find the cube root of 54854153 ?

Let $64000000 = \text{supposed cube}$, whose root is 400:

Then 64000000 $54^8 54^1 53$

128000000 100708206

54854153 64000000

As 182854153 : 173768306 :: 400

$$182854153)69483321400(379 = \text{root nearly}$$

Again, let $54439939 = \text{supposed cube}$, whose root is 379.

Then, 54439939 54854153

108879878

54854153

As 163734031 : 164148245 :: 279

379

1477334205

1149037715

492-44735

163734081)62212184855(379,958793+=
(Root correct)

SECOND

EXTRACTION of the CUBE ROOT.

SECOND METHOD by APPROXIMATION.

RULE 1.—Divide the resolvend by three times the assumed root, and reserve the quotient.

2. Subtract one twelfth part of the square of the assumed root from the quotient.

3. Extract the square root of the remainder.

4. To this root add one half of the assumed root, and the sum will be the true root, or an approximation to it;—Take this approximation as the assumed root, and, by repeating the process, a root farther approximated will be found, which operation may be farther repeated as often as necessary, and the root discovered to any assigned exactness.

Note. In order to find the value of the first assumed root, in this or any other power, divide the resolvend into periods by beginning at the place of units, and including in each period, so many figures as there are units in the exponent of the root; viz. 3 figures in the cube root;—4 for the biquadrate, and so on: Then by a table of powers, or otherwise, find a figure, which (being involved to the power whose exponent is the same with that of the required root) is the nearest to the value of the first period of the resolvend at the left hand; and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root: And it is of no importance whether the figure thus chosen, be, when involved, greater or less than the left hand period.

1. What is the cube root of 436036824287?

7000 = assumed root.

3

21,000) 436036824287 (20762658,2994
Subtr. $7000 \times 7000 \div 12 = 4083333.3333$

$\sqrt{166803249661} = 4084,15$
Add $\frac{1}{2}$ the assumed root, = 3500

And it gives the approximated root = 7584,15.

For

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For the second operation, use the approximated root as the assumed one, and proceed as above.

THIRD METHOD by APPROXIMATION.

1. Assume the root in the usual way, then multiply the square of the assumed root by 3, and divide the resolvent by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and, proceeding in this manner, the root may be extracted to any assigned exactness.

What is the cube root of 7?

Let the assumed root be 2. Then $2 \times 2 \times 3 = 12$ the divisor.

$12 \overline{) 7,06583}$, to this add $\frac{2}{3}$ of 2 = 1,333, &c. that is, $583 + 1,333 = 1,916$ approximated root.

Now assume 1,916 for the root, then, by the second process, the root is $\frac{7}{3 \times 1,916} = 2 + \frac{2}{3} \times 1,916 = 1,9126$, &c.

APPLICATION and USE of the CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

RULE 1.—Divide the *greater* by the *less*, and extract the cube root of the quotient.

2. Multiply the root, so found, by the *least* of the given numbers, and the product will be the *last*.

3. Multiply this product by the *same* root, and it will give the *greatest*.

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EXAMPLES.

1. What are the two mean proportionals between 6 and 750?

$750 \div 6 = 125$, and $\sqrt[3]{125} = 5$. Then $5 \times 6 = 30 =$ least, and $30 \times 5 = 150 =$ greatest. *Ans.* 30 and 150.

Proof. As $6 : 30 :: 150 : 750$.

2. What are the two mean proportionals between 56 and 19208?

Ans. 392 and 2744.

Note. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3. If a bullet 6 inches diameter weigh 32lb, What will a bullet of the same metal weigh, whose diameter is 3 inches?

$6 \times 6 \times 6 = 216$. $3 \times 3 \times 3 = 27$. As $216 : 32lb :: 27 : 4lb$, *Ans.*

4. If a globe of silver of 3 inches diameter, be worth £45, what is the value of another globe of a foot diameter?

$3 \times 3 \times 3 = 27$. $12 \times 12 \times 12 = 1728$. As $27 : 45 :: 1728 : £2880$ *Ans.*

The side of a cube being given, to find the side of that cube which shall be double, triple, &c. in quantity to the given cube.

RULE.—Cube your given side, and multiply it by the given proportion between the given and required cube, and the cube root of the product will be the side sought.

5. If a cube of silver, whose side is 4 inches, be worth £60, I demand the side of a cube of the like silver, whose value shall be 4 times as much?

$4 \times 4 \times 4 = 64$, & $64 \times 4 = 256$. $\sqrt[3]{256} = 6,349+$ inches, *Ans.*

6. There is a cubical vessel, whose side is 2 feet : I demand the side of a vessel, which shall contain three times as much?

$2 \times 2 \times 2 = 8$, and $8 \times 3 = 24$. $\sqrt[3]{24} = 2,884 = 2$ feet, $10\frac{2}{3}$ inches, *Ans.*

7. The

EXTRACTION of the BIQUADRATE ROOT. 135

7. The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches; I demand the side of a cubic box, which shall contain that quantity?

Ans. 12,908 inches.

8. Suppose a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold; What are the dimensions of a ship of 200 tons, of the same mould and shape?

$$89 \times 89 \times 89 = 704969 = \text{cubed keel.}$$

As $500 : 200 :: 704969 : 281987,6$ cube of the required keel.

$$\sqrt[3]{281987,6} = 65,57 \text{ feet the required keel.}$$

As $89 : 65,57 :: 36 : 26,52 = 6\frac{1}{2}$ feet, beam, nearly.

As $89 : 65,57 :: 16 : 11,7$ feet, depth of the hold.

EXTRACTION of the BIQUADRATE ROOT.

RULE.—Extract the square root of the resolvend, and then, the square root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

$\begin{array}{r} 20736(144 \\ 2 \\ \hline 24)107 \\ 96 \\ \hline 284)1136 \\ 1136 \\ \hline \end{array}$	$\begin{array}{r} 144(12 \text{ root required.} \\ 1 \\ \hline 22)44 \\ 44 \\ \hline \end{array}$
---	---

By APPROXIMATION.

METHOD 1.

1. Divide the resolvend by six times the square of the assumed root, and from the quotient subtract $\frac{1}{6}$ part of the square of the assumed root.
2. Extract the square root of the remainder.

3. Add

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3. Add $\frac{2}{3}$ of the assumed root to the square root, and the sum will be the true root, or an approximation to it.

4. For every succeeding operation (either in this or in the following method) proceed in the same manner as in the first, each time using the last approximated root for the assumed root.

The biquadrate root of 20736 is required.

Here, 10 is the assumed Root:

$$10 \times 10 \times 6 = 600 \quad 20736 \quad (34,56$$

$$\text{Subtract, } 10 \times 10 \div 18 = 5,5555$$

$$\sqrt{29,0044} = 5,38$$

$$\text{Add } \frac{2}{3} \text{ of } 10 = 6,66$$

Approximated root 12,04, to be made the assumed root for the next operation.

METHOD 2.

Divide the resolvent by four times the cube of the assumed root: To the quotient, add three fourths of the assumed root, and the sum will be the true root, or an approximation to it.

Let the biquadrate of 20736 be required, as before.

The assumed root is 10:

$$10 \times 10 \times 10 \times 4 = 4000 \quad 20736 \quad (5,184$$

$$\text{Add } \frac{3}{4} \text{ of } 10 = 7,5$$

Approximated root 12,684, to be made the assumed root for the next operation.

EXTRACTION of the SURSOLID ROOT, By APPROXIMATION.

A particular R U L E.

1. Divide the resolvent by five times the assumed root, and to the quotient add one twentieth part of the fourth power of the same root.

2. From

2. From the square root of this sum subtract one fourth part of the square of the assumed root.

3. To the square root of the remainder add one half of the assumed root, and the sum is the root required, or an approximation to it.

Note. This Rule will give the root true to five places, at the least, (and generally to eight or nine places) at the first process.

Required the sursolid Root of 281950621875 ?

200 = assumed root.

5

1000) 281950621875 quotient.

Add $200 \times 200 \times 200 \times 200 \div 20 = 8000000$

Subtract, $\sqrt{361950621875} = 19025$ nearly.
 $200 \times 200 \div 4 = 10000$

$\sqrt{9025} = 95$

Add half the assumed root = 100

Root 195 required.

A General RULE for extracting the ROOTS of all POWERS.

RULE * 1.—Prepare the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. Find

* The extracting of roots of very high powers will, by this rule, be a tedious operation :—The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more indices, whose product is equal to the given index, and extract out of the given number a root answering to another of the indices, and so on, to the last.

Thus, the fourth root = square root of the square root ; the sixth root = square root of the cube root ;—the eighth root = square root of the fourth root :—the ninth root = the cube root of the cube root ;—the tenth root = square root of the fifth root ;—the twelfth root = cube root of the fourth, &c.

2. Find the first figure of the root by trial, or by inspection into the Table of Powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend*.

4. Involve the root to the next inferior power to that which is given, and multiply it by the number, denoting the given power, for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number* as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

EXAMPLES.

1. What is the cube root of 20346417?

$$\begin{array}{r} 20346417(273 \\ 8 \quad = 1^{\text{st}} \text{ Subtrahend.} \end{array}$$

$$2 \times 3 = 12 \quad 123 = \text{Dividend.}$$

$$27 \quad = \quad 19683 = 2^{\text{d}} \text{ Subtrahend.}$$

$$27 \times 3 = 2187, 6634 = 2^{\text{d}} \text{ Dividend.}$$

$$273^3 = 20346417 = 3^{\text{d}} \text{ Subtrahend.}$$

.....
The Method of Operation.

$2 \times 2 \times 2 = 8$ root of the first period, or 1st subtrahend.

$2 \times 2 = 4$ (=next inferior power) and,

4×3

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4×3 (the index of the given power) $= 12$, 1st divif.

$27 \times 27 \times 27 = 19683 = 2d$ fubtrahend.

$27 \times 27 = 229$ (next inferior power) and,

729×3 (= index of the given power) $= 2187 = 2d$ div.

$273 \times 273 \times 273 = 20346417 = 3d$ fubtrahend.

2. What is the biquadrate root of 34827998976?

Ans. 431,9+.

OF PROPORTION IN GENERAL.

Numbers are compared together to discover the relations they have to each other.

There must be two numbers to form a comparison: The number, which is compared, being written first, is called the *antecedent*; and that, to which it is compared, the *consequent*.

Numbers are compared with each other two different ways: The one comparison considers the *difference* of the two numbers, and is called Arithmetical Relation, the difference being sometimes named the Arithmetical Ratio; and the other considers their *quotient*, which is termed Geometrical Relation, and the quotient, the Geometrical Ratio. Thus, of the numbers 12 and 4; the difference, or Arithmetical Ratio, is $12 - 4 = 8$; and the Geometrical

12
Ratio is $\frac{12}{4} = 3$.

4

If two or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms similarly profited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So, the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals;

*RATIOS are, here, always considered as the result of the greater term of comparison diminished, or divided, by the less; not regarding which of them be the antecedent.

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als; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.*

Proportion is distinguished into continual and discontinual. If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continual, and the numbers themselves, a series of continual proportionals, or an arithmetical or geometrical proportion. So 2, 4, 6, 8 form an arithmetical proportion; for $4-2=6-4=8-6=2$; and 2, 4, 8, 16, a geometrical proportion; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

Note. It is common to read the geometricals 2 : 4 :: 8 : 16 : Thus, 2 is to 4 as 8 to 16, or As 2 to 4 so is 8 to 16.

But if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinual. So 4, 2, 8, 6, are in discontinued arithmetical proportion; for $4-2=8-6=2$ =common difference of the couplets, $8-2=6$ =difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8 are in discontinued

ed geometrical proportion : For $\frac{4}{2}=\frac{16}{8}=2$

common ratio of the couplets, and $\frac{16}{2}=8$ =ratio of the consequent of one couplet and the antecedent of the next.

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* Four numbers are said to be *reciprocally* or *inversely* proportional, when the fourth is less than the second, by as many times as the third is greater than the first, or when the first to the third, is as the fourth to the second, and *vice versa*. Thus 2, 9, 6 and 3 are reciprocal proportionals.

ARITHMETICAL PROPORTION. 245

ARITHMETICAL PROPORTION.

THEOREM 1.—If any four quantities, 2, 4, 6, 8, be in Arithmetical Proportion, the sum of the two means is equal to the sum of the two extremes.

And if any three quantities, 2, 4, 6, be in Arithmetical Proportion, the double of the mean is equal to the sum of the extremes.

THEOREM 2.—In any continued arithmetical proportion, 1, 3, 5, 7, 9, 11, the sum of the two extremes, and that of every other two terms, equally distant from them, are equal: Thus, $1+11=8+9=5+7$.

When the number of terms is odd, as in the proportion 2, 8, 12, 18, 22, then, the sum of the two extremes being double to the mean, or middle term, the sum of any other two terms, equally remote from the extremes, must likewise be double to the mean.

THEOREM 3.—In any continued arithmetical proportion, $4, 4+2, 4+4, 4+6, 4+8$, &c. the last or greatest term is equal to the first or least more the common difference of the terms drawn into the number of all the terms after the first, or into the whole number of the terms, less one.

THEOREM 4.—The sum of any rank, or series, of quantities in continued arithmetical proportion, 1, 3, 5, 7, 9, 11, is equal to the sum of the two extremes multiplied into half the number of terms.*

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* The same thing also holds, when the number of terms is odd, as in the series 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the sum of any two terms, equally distant from it on contrary sides, it is obvious that the value of the whole series is the same as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the sum of the two extremes) multiplied by the whole number of terms; or to the sum of the extremes multiplied by half the number of terms.

The sum of any number of terms of the arithmetical series of odd numbers, 1, 3, 5, 7, 9, &c. is equal to the square of the number.

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ARITHMETICAL PROGRESSION.

Any rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series or progression; if the contrary, a descending series.

So { 0. 1. 2. 4. 6. 8. &c. is an ascending arithmetical series.
 { 1. 2. 4. 8. 16. 32. &c. is an ascending geometrical series,
 & { 8. 6. 4. 2. 1. 0. &c. is a descending arithmetical series,
 { 32. 16. 8. 4. 2. 1. &c. is a descending geometrical series.

The numbers which form the series, are called the terms of the progression.

Note. The first and last terms of a progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM 1. *The first term, the last term, and the number of terms being given, to find the common difference.*

RULE.

For, $0+1$ or the sum of 1 term $= 1^2$ or 1

$1+3$ or the sum of 2 terms $= 2^2$ or 4

$4+5$ or the sum of 3 terms $= 3^2$ or 9

$9+7$ or the sum of 4 terms $= 4^2$ or 16

$16+9$ or the sum of 5 terms $= 5^2$ or 25, &c.

Whence, it is plain, that let there be any number whatever, the sum of its terms will be equal to its square.

EXAMPLES.

* The first term, the ratio, and number of terms given, to find the sum of the series.

A Gentleman travelled 29 days; the first day he went but 4 miles, and increased every day's travel 2 miles: How far did he travel?

$29 \times 29 = 841$ miles, the Answer.

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RULE.—Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

EXAMPLES.

1. The extremes are 9 and 39, and the number of terms is 19; What is the common difference?

$$\begin{array}{r} 39 \\ - 9 \\ \hline \end{array} \left. \begin{array}{l} \text{Ex-} \\ \text{tremes.} \end{array} \right\}$$

Divide by the number of terms less 1 = $19 - 1 = 18$) 36 (2 Answer.

$$\text{Or, } \frac{39 - 9}{19 - 1} = 2.$$

2. A man had 10 sons, whose several ages differed alike; the youngest was 9 years old, and the eldest 48; What was the common difference of their ages?

$$\text{Ans. } \frac{48 - 9}{10 - 1} = 5.$$

3. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles; required the daily increase?

$$\frac{37 - 5}{9 - 1} = 4 \text{ the Ans.}$$

PROBLEM 2. *The first term, the last term, and the number of terms being given, to find the sum of all the terms.*

RULE.—Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

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EXAMPLES.

1. The extremes of an arithmetical series are 3 and 39, and the number of terms 19; required the sum of the series?

$$\begin{array}{r} 39 \\ + 3 \\ \hline \end{array} \left. \begin{array}{l} \\ \end{array} \right\} \text{Extremes.}$$

$$\text{Sum} = 42$$

$$\text{Number of terms} = \times 19$$

$$\begin{array}{r} 378 \\ 42 \\ \hline \end{array}$$

$$\begin{array}{r} 42 \\ \hline \end{array}$$

$$\begin{array}{r} 8178 \\ \hline \end{array}$$

$$\text{Or, } \frac{39+3}{2} \times 19 = 399$$

399 the Answer.

2. It is required to find how many strokes the hammer of a clock would strike in a week, or 168 hours, provided it increased 1 at each hour?

$$\frac{168+1 \times 168}{2} = 14196 \text{ Ans.}$$

3. Suppose a number of stones were laid a yard distant from each other for the space of a mile, and the first, a yard from a basket; what length of ground will that man travel over, who gathers them up singly, returning with them one by one to the basket?

$$\frac{3520+2 \times 1760}{2} = 3099360 \text{ yds.} = 1761 \text{ miles, Ans.}$$

N. B. In this question, there being 1760 yards in a mile, and the man returning with each stone to the

ARITHMETICAL PROGRESSION. 245

the basket, his travel will be doubled; therefore the first term will be 2, and the last 1760×2 , and the number of terms 1760.

4. A man bought 25 yards of linen in Arithmetical Progression; for the 4th yard he gave 12 shillings, and for the last yard £3 15s.—What did the whole amount to, and, what did it average per yard?

$$\begin{array}{r} 75 - 12 \\ \hline 22 - 1 \end{array} = 9 \text{ the common difference, by which the (first term is found to be 3.}$$

Then, $\frac{75 + 3 \times 25}{2} = £48.15s.$ and the average price (is £1.19s. per yard.

5. Required the sum of the first 1000 numbers in their natural order?

$$\frac{1000 + 1 \times 1000}{2} = 500500 \text{ Answer}$$

PROBLEM 3. Given the extremes and the common difference, to find the number of terms.

RULE.—Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

EXAMPLES.

1. The extremes are 3 and 39, and the common difference 2; what is the number of terms?

$$\begin{array}{r} 39 \\ - 3 \\ \hline 36 \\ \div 2 \\ \hline 18 \end{array} \left. \begin{array}{l} 39 \\ 3 \end{array} \right\} \text{Extremes.}$$

Common difference = 2) 36.

$$\begin{array}{r} \text{Quotient} = 18 \\ \text{Add } 1 \\ \hline 19 \end{array}$$

19 the Answer.

Or, $\frac{39 - 3}{2} + 1 = 19$

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2. A man, going a journey, travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles; How many days did he travel, and, how far?

$$\frac{51-7}{4}+1=12 \text{ days, and } \frac{51+7 \times 12}{2}=348 \text{ miles, Ans.}$$

PROBLEM 4. *The extremes and common difference given, to find the sum of all the series.*

RULE.—Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference, will give the sum.

EXAMPLES.

1. If the extremes are 3 and 39, and the common difference 2; what is the sum of the series?

$$39+3=42=\text{Sum of the extremes.}$$

$$39-3=36=\text{Difference of extremes.}$$

$$36+2=38=\text{Difference of extremes increased by (the common difference)}$$

$$\begin{array}{r} 42 \\ \times 38 \\ \hline \end{array}$$

$$1596$$

$$\text{Twice the common diff.} = 4 \overline{)1596}$$

$$399 \text{ the Answer.}$$

$$\text{Or, } \frac{39+3 \times 39-3+2}{2 \times 2} = 399$$

3. A owes B a certain sum, to be discharged in a year, by paying 6d. the first week, 12d. the second, and thus to increase every weekly payment by a shilling,

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shilling, till the last payment be £2 11s. 6d. what is the debt?

$$\frac{5 \cdot 5 + 5 \times 5 \cdot 5 - 5 + 1}{1 \times 3} = £67 \text{ 12s. Ans.}$$

PROBLEM 5. *The extremes and the sum of the series given, to find the common difference.*

RULE.—Divide the product of the sum and difference of the extremes, by the difference of twice the sum of the series, and the sum of the extremes, and the quotient will be the common difference.

EXAMPLE.

Let the extremes be 3 and 39, and the sum 399: what is the common difference?

$$\text{Sum of the extremes} = 39 + 3 = 42$$

$$\text{Diff. of the extremes} = 39 - 3 = 36$$

$$42 \times 36 = 1512$$

$$399 \times 2 - 42 = 756$$

$$1512 \div 756 = 2 \text{ the Ans.}$$

$$\frac{3 + 39 \times 39}{2} = 789$$

$$\text{Or, } 399 \times 2 - 39 + 3 = 756$$

$$756 \div 36 = 21 \text{ the Ans.}$$

PROBLEM 6. *The extremes and sum of the series given, to find the number of terms.*

RULE.—Twice the sum of the series, divided by the sum of the extremes, will give the number of terms.

EXAMPLE.

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EXAMPLE.

Let the extremes be 8 and 39, and the sum of the series 399; what is the number of terms?

Sum of the series = 399

Sum of the extr. = $89 + 8 = 42$ 798 (19 the Ans.)

$$\text{Or } \frac{399 \times 2}{39 + 8} = 19.$$

$$\begin{array}{r} 378 \\ 378 \\ \hline \end{array}$$

GEOMETRICAL PROPORTION.

THEOREM 1.—If four quantities, 2, 6, 4, 12, be in geometrical proportion, the product of the two means, 6×4 , will be equal to that of the two extremes, 2×12 , whether they are continued, or discontinued; and, if three quantities, 2, 4, 8, the square of the mean is equal to the product of the two extremes.

THEOREM 2.—If four quantities, 2, 6, 4, 12, are such that the product of two of them, 2×12 , is equal to the product of the other two, 6×4 , then are those quantities proportional.

THEOREM 3.—If four quantities, 2, 6, 4, 12, are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.

THEOREM 4.—The products of the corresponding terms of two geometrical proportions are also proportional.

That is, if $2 : 6 :: 4 : 12$ and $2 : 4 :: 5 : 10$, then will $2 \times 2 : 6 \times 4 :: 4 \times 5 : 12 \times 10$.*

THEOREM.

* And if any quantities be proportional, their squares, cubes, &c. will likewise be proportional.

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THEOREM 5.—If four quantities, 2, 6, 4, 12, are directly proportional.

Then, {	1. Directly,	$2 : 6 :: 4 : 12$
	2. Inversely,	$6 : 2 :: 12 : 4$
	3. Alternately,	$2 : 4 :: 6 : 12$
	4. Compoundly,	$2 : 8 :: 4 : 16$
	5. Dividedly,	$2 : 4 :: 4 : 8$
	6. Mixtly,	$8 : 4 :: 16 : 8$
	7. By Multiplication,	$2r : 6r :: 4 : 12$ $2 \quad 6$
	8. By Division,	$\frac{2}{r} : \frac{6}{r} :: 4 : 12$ $r \quad r$

Because the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 2.

THEOREM 6.—If three numbers, 2, 4, 8, be in continued proportion, the square of the first will be to that of the second, as the first number to the third; that is, $2 \times 2 : 4 \times 4 :: 2 : 8$.

THEOREM 7.—In any continued geometrical proportion (1, 3, 9, 27, 81, &c.) the product of the two extremes, and that of every other two terms, equally distant from them, are equal.

THEOREM 8.—The sum of any number of quantities, in continued geometrical proportion, is equal to the difference of the rectangle of the second and last terms, and the square of the first divided by the difference of the first and second terms.

GEOMETRICAL PROGRESSION.

A Geometrical Progression is when a Rank or Series, of numbers, increases, or decreases, by the continual multiplication, or division, of some equal number.

PROBLEM 1. Given one of the extremes, the ratio, and the number of the terms of a Geometrical Series, to find the other extreme.

RULE.—Multiply, or divide (as the case may require) the given extreme by such power of the ratio, whose exponent

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exponent* is equal to the number of terms less 1, and the product, or quotient, will be the other extreme.

EXAMPLES.

1. If the first term be 4, the ratio 4, and the number of terms 9; what is the last term?

$$2. \quad 2. \quad 3. \quad 4. \quad + \quad 4 = \quad 8$$

4. 16. 64. 256. $\times \quad 256 = 65536 =$ Power of the ratio, whose exponent is less by 1, than the number of terms.

$$65536 = 8^{\text{th}} \text{ power of the ratio,}$$

$$\text{Multiply by } \quad 4 = \text{first term.}$$

$$262144 = \text{last term.}$$

$$\text{Or, } 4 \times 4^8 = 262144 = \text{the Ans.}$$

* As the last term, or any term near the last, is very tedious to be found by continual multiplication, it will often be necessary, in order to ascertain them, to have a series of numbers in Arithmetical Proportion, called *Indices*, or *Exponents*, beginning either with a cypher, or an unit, whose common difference is one.

When the *first* term of the series and the *ratio*, are equal, the *indices* must begin with an unit; and, in this case, the product of any two terms is equal to that term, signified by the sum of their *indices*.

Thus, $\begin{cases} 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \quad \&c. \text{ Indices, or Arithmetical series.} \\ 2. \quad 4. \quad 8. \quad 16. \quad 32. \quad 64. \quad \&c. \text{ Geometrical series (leading term).} \end{cases}$

Now, $6 + 6 = 12 =$ the Index of the twelfth term, and

$$64 \times 64 = 4096 = \text{the twelfth term.}$$

But when the *first* term of the series and the *ratio* are different, the *indices* must begin with a cypher, and the sum of the *indices*, made choice of, must be one less than the number of terms, given in the question; because 1 in the *indices* stands over the second term, and 2 in the *indices* over the third term, &c. And, in this case, the product of any two terms, divided by the first, is equal to that term beyond the first, signified by the sum of their *indices*.

Thus, $\begin{cases} 0. \quad 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \quad \&c. \text{ Indices.} \\ 1. \quad 3. \quad 9. \quad 27. \quad 81. \quad 243. \quad 729. \quad \&c. \text{ Geometrical series.} \end{cases}$

Here, $6 + 5 = \quad 11$ the Index of the 12th term,

$729 \times 243 = 177147$ the 12th term, because the first term of the series and the ratio are different, by which mean a cypher stands over the first term,

Thus, by the help of these *indices*, and a few of the first terms in any geometrical series, any term, whose distance from the first term is assigned, though it were ever so remote, may be obtained without producing all the terms.

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2. If the last term be 262144, the ratio, 4, and the number of terms 9 ; what is the first term ?

8th power of the ratio $4^8 = 65536$) 262144 (4 = the first term.

Or, $\frac{262144}{4^8} = 4$ the first term.

Again, Given the first term, and the ratio, to find any other term assigned.

RULE 1.—When the indices begin with an unit.

1. Write down a few of the leading terms of the series, and place their indices over them.

2. Add together such indices, whose sum shall make up the entire index to the term required.

3. Multiply the terms of the geometrical series, belonging to those indices, together, and the product will be the term sought.

1. If the first term be 1, and the ratio 2 ; what is the 13th term ?

1. 2. 3. 4. 5 + 5 + 3 = 13

1. 4. 8. 16. 32 X 32 X 8 = 8192 *Ans.*
Or, $1 \times 2^{12} = 8192$.

2. A merchant wanting to purchase a cargo of horses for the Westindies, a jockey told him he would make all the trouble and expense, upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to by doubling the whole number by a half penny, that is, two farthings for the first, four for the second, eight for the third, &c. to which the merchant, thinking he had made a very good bargain, readily agreed : Pray, what did the last horse come

Note. If the ratio of any geometrical series be double, the difference of the greatest and least terms is equal to the sum of all the terms, except the greatest : If the ratio be triple, the difference is double the sum of all but the greatest : If the ratio be quadruple, the difference is triple the sum of all but the greatest, &c. In any geometrical series decreasing, and continued ad infinitum, the greatest term is equal to the sum of all the remaining terms ad infinitum.

come to ; and, what did the horses, one with another, cost the merchant ?

[30th, or last term,
 1. 2. 3. 4. 5. $6 + 6 = 12th$, $12 + 12 + 6 =$
 2. 4. 8. 16. 32. $64 \times 64 = 4096$ $4096 \times 4096 \times 64 =$
 $1073741824 qrs. = \pounds 11,848$ 15. 4d. and their average price was $\pounds 37232$ 14s. 0½d. a piece.

RULE 2.—When the Indices begin with a cypher,

1. Write down a few of the leading terms of the series, as before, and place their indices over them.

2. Add together the most convenient indices to make an Index, less by 1 than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series, together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power, whose Index is one less than the number of terms multiplied, and make the result a divisor, by which, divide the dividend, and the quotient will be that term beyond the first, signified by the sum of those indices, or the term sought.

3. If the first term be 5, and the ratio 3, What is the 7th term ?

Q. 1. 2. $3 + 2 + 1 =$ 6 = Ind. to the term
 (beyond 1st, or 7th.

5. 15. 45. $135 \times 45 \times 15 = 91125 =$ Dividend.

The number of terms, multiplied, is 3 (viz. $135 \times 45 \times 15$) and $3 - 1 = 2$, is the power to which the term 5 is to be raised ; but the 2d power of 5 is $5 \times 5 = 25$, and therefore $91125 \div 25 = 3645$ the 7th term required.

PROB. 2. Given the first term, the ratio, and number of terms, to find the sum of the series.

RULE.—Raise the ratio to a power, whose Index shall be equal to the number of terms, from which subtract 1 ; divide the remainder by the ratio, less 1, and

GEOMETRICAL PROGRESSION. 253

1, and the quotient, multiplied by the first term, will give the sum of the series.

EXAMPLES.

1. If the first term be 5, the ratio 3, and the number of terms 7; What is the sum of the series?

$$\text{Ratio} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187 = 7^{\text{th}} \text{ power}$$

Subtract 1 (of the ratio.

$$\text{Divide by the ratio less 1} = \frac{2187 - 1}{3 - 1} = 1093$$

$$\text{Quotient} = 1093$$

$$\text{Multiply by the first term} = 5465$$

$$\text{Sum of the series} = 5465$$

$$\text{Or, } \frac{3^7 - 1}{3 - 1} \times 5 = 5465 \text{ Answer.}$$

2. A shopkeeper sold 13 yards of cloth, on the following terms, viz. 2d. for the first yard, 4d. for the second, 8d. for the third, &c. I demand the price of the cloth.

$$\frac{2^{13} - 1}{2 - 1} \times 2 = 16382d. = \pounds 68 \text{ } 5s. \text{ } 2d. \text{ Answer.}$$

3. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; Pray what did her portion amount to?

$$\frac{3^{12} - 1}{3 - 1} \times 1 = 265720 \text{ guineas, Answer.}$$

4. What debt can be discharged in a year, by paying 1 shilling the first month, 10s. the second, and so on, each month in a tenfold proportion?

$$\frac{10^{12} - 1}{10 - 1} \times 1 = 111111111111 \text{ shillings,} \\ \pounds 55555555555 \text{ } 11s. \text{ Answer.}$$

5. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on in an eight fold proportion ; I demand what his 9 days' labour amounted to, rating the wheat at 5s. per bushel ?*

$$\text{Ans. } \frac{8^9 - 1}{8 - 1} \times 8 = 152391688 \text{ corns. Amount} = \text{£}78 \text{ os. } (5\frac{1}{2}d.)$$

6. An ignorant fop wanting to purchase an elegant house ; a facet ous gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a penny for the first door, 2d. for the second, 4d. for the third, and so on, doubling at every door, which were 36 in all : It is a bargain, cried the simpleton, and here is a guinea to bind it ; Pray what would the house have cost him ?

$$\text{Ans. } \frac{2^{36} - 1}{2 - 1} \times 1 = 68719476735d. = \text{£}286331153 \text{ 1s. } 3d.$$

7. A young fellow, well skilled in numbers, agreed with a rich farmer to serve him 10 years, without any other reward, but the produce of one wheat corn for the first year, and that produce to be sowed from year to year, till the end of the time, allowing the increase but in a tenfold proportion ; What is the sum of the whole produce ? and, what will it amount to, at 5s. per bushel ?

$$\text{Ans. } \frac{10^{10} - 1}{10 - 1} \times 10 = 11111111110 \text{ corns. Amount} = (\text{£}56518s. 6\frac{1}{2}d.)$$

8. Suppose one farthing had been put out at 6 per cent. per annum, compound interest,† at the commencement

* Note, 7680 wheat or barley corns are supposed to make a pint.
† Any sum, at 6 per cent. per annum, compound interest, will double in eleven years and three hundred and twenty five days, or 11.89 years, or 11.89 is near enough ; then, if you divide 1784 by 11.89, it will give the number of terms in this case equal to 150 ; — The ratio will be 2, and the first term 1.

GEOMETRICAL PROGRESSION. 255

menacement of the Christian Æra : What would it have amounted to in 1784 years ; and suppose the amount to be in standard gold, allowing a cubic inch to be worth £53 2s. 8d. how large would the mass have been ?

$$\text{Ans. } \frac{2^{150}-1}{2-1} \times 1 = £14867163465687482094357145$$

$$(51509850767065361115. 3\frac{1}{2}d.)$$

$$= 27980859722121230415979571232933594210766 \text{ cubic inch-}$$

$$\text{(es of gold.)}$$

As, 355 : 113 :: 360 × 69,5 : 7964 Earth's diameter.
 360 × 69,5 × 7964 × 1327,33 = 264482820122 cubic miles in the globe = 67273337308854741368832000 cubic inches in the globe.
 Then, 27980859722121230415979571232933594210766 ÷ 67273337308854741368832000 = 41593089984288,8 which, however incredible it may appear to some, is more than four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety nine millions, eight hundred and forty thousand, two hundred and eighty eight times larger than the globe we inhabit.

PROBLEM 3. *The first term, the last term (or the extremes) and the ratio given, to find the sum of the series.*

RULE.—Divide the difference of the extremes by the ratio less by 1 : Add the greater extreme to the quotient, and the result will be the sum of all the terms.

Or, multiply the greatest term by the ratio, from the product subtract the least term ; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

Or, when all the terms are given, then, from the product of the second and last terms subtract the square of the first term ; this remainder being divided by the second term less the first, will give the sum of the series.

EXAMPLES.

1. If the series be 2. 6. 18. 54. 162. 486. 1458. 4374 ; What is its sum total ?

First

256 GEOMETRICAL PROGRESSION.

First Method.

From the greatest term = 4374

Subtract the least = 2

Divide by the ratio, less 1 = $3 - 1 = 2$) 4372 diff. of ext.

Quotient = 2186

Add the greater extreme = 4374

6560

$$\text{Or, } \frac{4374 - 2}{3 - 1} + 4374 = 6560 \text{ Ans.}$$

Second Method.

Greatest term = 4374

Multiply by the ratio = 3

Product = 13122

Subtract the least term = 2

Divide by the ratio, less by $1 \times 3 - 1 = 2$) 13120

Ans. 6560

$$\text{Or, } \frac{4374 \times 3 - 2}{3 - 1} = 6560.$$

Third Method.

Greatest term = 4374

Multiply by the second term = 6

Product = 26244

Subtract the square of the first term = $2 \times 2 = 4$

Div. the rem. by the 2d term less the 1st = $6 - 2 = 4$) 26240

Ans. 6560

$$\text{Or, } \frac{4374 \times 6 - 4}{6 - 2} = 6560$$

GEOMETRICAL PROGRESSION. 257

2. A man travelled 6 days; the first day he went 4 miles, and each day doubling his day's travel; his last day's ride was 128 miles; How far did he go in the whole?

$$\frac{128-4}{2-1} + 128 = 252 \text{ miles, Ans.}$$

3. A gentleman dying, left 5 sons; to whom he bequeathed his estate as follows, viz. to his youngest son £1000; to the eldest £5062 10s. and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; What did the several legacies amount to?

$$\frac{5062.5-1000}{1.5-1} + 5062.5 = £13187 \text{ 10s. Ans.}$$

PROB. 4. Given the extremes and ratio, to find the number of terms.

RULE.—Divide the greatest term by the least; find what power of the ratio is equal to the quotient; then, add 1 to the index of that power, and the sum will be the number of terms.

Or, Subtract the logarithm* of the least term from that of the greatest; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

If the least term be 2, the greatest term 4374, and the ratio 3, What is the number of terms?

Divide by the least term $\frac{4374}{2} = 2187 = \text{greatest term}$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = \text{Quo. } 2187 = 7^{\text{th}} \text{ power, then } 7+1=8 \text{ the Ans.}$

* Logarithms are artificial numbers, the addition of, which answers to multiplication of whole numbers, and subtraction to division.

$O\tau,$

From the logarithm of
the greatest term = 8.64088
Subtract the logarithm
of the least term = 0.80103
Divide the re-
mainder by
the logarithm
of the ratio } = $\frac{7.83985}{.83984} = 9.33412$ Ans.

PROB. 5. Given the least term, the ratio, and the sum of the series, to find the last term.

RULE.—Multiply the sum of the series by the ratio, less 1; to that product add the first term, and the result, divided by the ratio, will give the last term.

E x A M P L E.

If the first term be 2, the ratio 3, and the sum of the series 6560 ; What is the last term ?

$$\frac{3-1 \times 6560 + 2}{3} = 4374 \text{ Ans.}$$

SIMPLE INTEREST.

Interest is a premium allowed by the borrower of any sum of money to the lender, according to a certain rate per cent. agreed on, which by law is stated at 6, that is, 6 for the use of £100 for one year.

Principal is the money lent.

Rate is the sum per cent. agreed on.

Amount is the principal and interest added together.

Interest is of two sorts, *simple* and *compound*.

Simple interest is that, which is allowed for the principal lent only.

Note. The rules for simple interest serve also to calculate commission, brokerage, insurance, purchasing stocks, or any thing else rated at so much per cent.

SIMPLE INTEREST. 259

GENERAL RULE.

1. Multiply the principal by the rate, and divide the product by 100 (or, which is the same, cut off the two right hand figures in the pounds, which must be reduced to the lowest denomination, each time cutting off as at first) and the quotient will be the answer for one year.

2. For more years than one, multiply the interest of one year by the given number of years, and the product will be the answer for that time.

3. If there be parts of a year, as months, or days, work for the months by the aliquot parts of a year, and for the days, by the Rule of Three Direct, or (which is sufficiently exact for common use, allowing 30 days to the month) take aliquot parts of the same.

Note, when the rate per cent. per annum, is $\left\{ \begin{array}{l} 9 \\ 8 \\ 6 \\ 4 \\ 3 \\ 2 \end{array} \right\}$ multiply the principal by $\left\{ \begin{array}{l} \frac{3}{4} \\ \frac{2}{5} \\ \frac{3}{5} \\ \frac{2}{5} \\ \frac{1}{2} \\ \frac{1}{5} \end{array} \right\}$ of the given number of months, cutting off, as before directed, and you will have the interest for the given time.

EXAMPLES.

1. What is the interest of £573 13 9½ for one year, at 6 per cent. per annum.

$$\begin{array}{r}
 \text{£. s. d.} \\
 573 \quad 13 \quad 9\frac{1}{2} \\
 \underline{6} \\
 34 \quad 42 \quad 2 \quad 9 \\
 \underline{20} \\
 8 \quad 42 \quad 8 \\
 \underline{12} \\
 5 \quad 13 \quad 1 \\
 \underline{4} \\
 9 \quad 52 \quad 4 \quad 2 \quad 5
 \end{array}$$

Ans. £84 8 5

2. What

260 SIMPLE INTEREST.

2. What is the
interest of £329 17
6½ for 3 years, 7
months and 12
days, at £5 per
cent. per annum?

£. s. d.

329 17 6½

5

16 49 7 8½

20

9 87

12

10 52

4

2 10

6 months ½

16 9 10½

Then,
interest of 1 year

3

1 month ⅓

49 9 7½

ditto of 3 years

10 days ⅒

8 4 11½

ditto of 6 months

2 days ⅙

1 7 5½

ditto of 1 month

9 1½

ditto of 10 days

1 9½

ditto of 2 days

Y. m. d.

£59 13

ditto of 3 7 12

Or thus :

£. s. d.

329 17 6½

£5

10

6 months ½

16 9 10½

8

1 ditto ⅓

49 9 7½

8 4 11½

10 days ⅒

1 7 5½

2 days ⅙

9 1½

1 9½

£59 13 4½

3. Wh

SIMPLE INTEREST. 26r

3. What is the interest of £439 12 9½ at £6 per cent. per annum, for 16 months?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 439 \quad 12 \quad 9\frac{1}{2} \end{array}$$

8 = ½ the number of months.

$$\text{Ans. } \text{£} 35 | 17 \quad 2 \quad 4$$

$$\text{s. } 3 | 42$$

$$\text{d. } 5 | 08$$

4. What is the interest of £591 15 9½ for 15 months, at £8 per cent. per annum?

$$\text{£} 591 \quad 15 \quad 9\frac{1}{2}$$

10 = ⅔ the number of months.

$$\text{£} 59 | 17 \quad 17 \quad 8\frac{1}{2}$$

$$\text{s. } 3 | 57$$

$$\text{d. } 6 | 92$$

$$\text{qr. } 8 | 70$$

5. What is the interest of £347 7 5½ at £4 per cent. per annum, for 15 months?

$$\text{£} 347 \quad 7 \quad 5\frac{1}{2}$$

5 = ⅓ the number of months.

$$\text{£} 17 | 36 \quad 17 \quad 3\frac{1}{2}$$

$$\text{s. } 7 | 87$$

$$\text{d. } 4 | 47$$

$$\text{qr. } 1 | 90$$

6. What

252 SIMPLE INTEREST.

6. What is the interest of £517 15 4 for one month, at £6 per cent. per annum?

Ans. £2 11 9½

7. Of £457 12 8½ for 2 months, at 6 per cent. per annum?

Ans. £4 11 6½

8. Of £347 5 9 for 3 months, at £6 per cent. per annum?

Ans. £5 4 2½

9. Of £397 19 for 4 months, at £6 per cent. per annum?

Ans. £7 19 5½

10. £508 10 5½ for 5 months, at £6 per cent. per annum?

Ans. £12 14 3½

11. £719 19 4 for six months, at £6 per cent. per annum?

Ans. 21 11 11½

12. £396 5 10 for 7 months, at £6 per cent. per annum?

Ans. £13 17 4½

13. £517 11 11½ for 8 months, at £6 per cent. per annum?

Ans. 20 14 0½

14. £245 5 8½ for 9 months, at £6 per cent. per annum?

Ans. £11 0 9

15. £195 15 5½ for 10 months, at £6 per cent. per annum?

Ans. £9 15 9½

16. £148 13 6½ for 11 months, at £6 per cent. per annum?

Ans. £8 3 5½

17. £509 9 2 for 12 months, at £6 per cent. per annum?

Ans. £33 2 3½

18. £317 17 8½ for 14 months, at £6 per cent. per annum?

Ans. £22 5 0½

19. £443 10 3 for 15 months, at £6 per cent. per annum?

Ans. £33 5 3

20. £293 7 9 for 16 months, at £6 per cent. per annum?

Ans. £23 9 5

21. £333 13 3¼ for 17 months, at £6 per cent. per annum?

Ans. £28 7 2½

22. £517 6 6 for 18 months, at £6 per cent. per annum?

Ans. £46 11 2

23. £347 13 7½ for 19 months, at £6 per cent. per annum?

Ans. 33 0 4½

24. £419 12 5 for 20 months, at £6 per cent. per annum?

Ans. £41 19 2½

25.

SIMPLE INTEREST BY DECIMALS. 253

25. £537 13 5½ for 16 months, at £9 per cent. per annum? *Ans.* £64 10 4½

26. £1 for 1 year, at 6 per cent. per annum? *Ans.* 1s. 2d. 1½gr.

SIMPLE INTEREST BY DECIMALS.

TABLE of RATIOS, from one pound, &c. to seven pounds.

Rate per Cent.	Ratios.	Rate per Cent.	Ratios.	Rate per Cent.	Ratios.
1	.01	3	.03	5	.05
1½	.0125	3½	.0375	5½	.0525
1¾	.015	3¾	.035	5¾	.055
1¾	.0175	3¾	.0375	5¾	.0575
2	.02	4	.04	6	.06
2½	.0225	4½	.0425	6½	.0625
2½	.025	4½	.045	6½	.065
2¾	.0275	4¾	.0475	6¾	.0675

Ratio is the simple interest of £1 for 1 year, at the rate per cent. agreed on.

TABLE for the ready finding of the decimal parts of a year, equal to any number of days, or quarters of a year.

Days	Decimal pts.	D.	Decimal pts.	Days	Decimal pts.
1	.00274	10	.027397	100	.273973
2	.005479	20	.054794	200	.547945
3	.008219	30	.082198	300	.821918
4	.010959	40	.109589	365	1.000000
5	.013699	50	.136986	$\frac{1}{4}$ of a year = .25 $\frac{1}{2}$ of a year = .5 $\frac{3}{4}$ of a year = .75	
6	.016438	60	.164383		
7	.019178	70	.191781		
8	.021918	80	.219178		
9	.024657	90	.246575		

When the principal, time and ratio are given, to find the interest, and amount.

RULE.—Multiply the principal, time and ratio continually together, and the last product will be the interest,

252 SIMPLE INTEREST.

6. What is the interest of £517 15 4 for one month, at £6 per cent. per annum?

Ans. £2 11 9½

7. Of £437 12 8½ for 2 months, at 6 per cent. per annum?

Ans. £4 11 6½

8. Of £347 5 9 for 3 months, at £6 per cent. per annum?

Ans. £5 4 2

9. Of £397 19 for 4 months, at £6 per cent. per annum?

Ans. £7 19 9

10. £508 10 5½ for 5 months, at £6 per cent. per annum?

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Ans. 21 11 11½

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Ans. 20 14 0½

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Ans. £9 15 9½

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Ans. £8 3 5½

17. £509 9 2 for 12 months, at £6 per cent. per annum?

Ans. £39 2 8½

18. £317 17 8½ for 14 months, at £6 per cent. per annum?

Ans. £22 5 0½

19. £443 10 3 for 15 months, at £6 per cent. per annum?

Ans. £33 5 8

20. £293 7 9 for 16 months, at £6 per cent. per annum?

Ans. £18 9 5

21. £338 13 3½ for 17 months, at £6 per cent. per annum?

Ans. £18 7 2½

22. £517 6 6 for 18 months, at £6 per cent. per annum?

Ans. £46 11 2

23. £347 13 7½ for 19 months, at £6 per cent. per annum?

Ans. 33 0 4½

24. £419 12 5 for 20 months, at £6 per cent. per annum?

Ans. £41 19 2½

SIMPLE INTEREST BY DECIMALS. 253

25. £587 12 5½ for 16 months, at £9 per cent. per annum ? *Ans.* £64 10 4½

26. £1 for 1 year, at 6 per cent. per annum ? *Ans.* 1s. 2d. 1½gr.

SIMPLE INTEREST BY DECIMALS.

TABLE OF RATIOS, from one pound, &c. to seven pounds.

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1¾	.015	3¾	.035	5¾	.055
1½	.0175	3¾	.0375	5¾	.0575
2	.02	4	.04	6	.06
2½	.0225	4½	.0425	6½	.0625
2¾	.025	4¾	.045	6¾	.065
2¾	.0275	4¾	.0475	6¾	.0675

Ratio is the simple interest of £1 for 1 year, at the rate per cent. agreed on.

TABLE for the ready finding of the decimal parts of a year, equal to any number of days, or quarters of a year.

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2	.005479	20	.054794	200	.547945
3	.008219	30	.082192	300	.821918
4	.010959	40	.109589	365	1.000000
5	.013699	50	.136986	$\frac{1}{4}$ of a year = .25 $\frac{1}{2}$ of a year = .5 $\frac{3}{4}$ of a year = .75	
6	.016438	60	.164383		
7	.019178	70	.191781		
8	.021918	80	.219178		
9	.024657	90	.246575		

When the principal, time and ratio are given, to find the interest, and amount.

RULE.—Multiply the principal, time and ratio continually together, and the last product will be the interest,

164 SIMPLE INTEREST BY DECIMALS.

interest, commission, brokerage, &c. to which add the principal, and the sum will be the amount,

EXAMPLES.

1. Required the amount of £537 10s. at £6 per cent. per annum, for 5 years?

Principal	537.5
Multiply by the ratio =	.06
Product	32.250
Multiply by the time =	5
Interest =	161.250
Add the principal =	537.5
Amount =	£698.75

Or, $537.5 \times .06 \times 5 + 537.5 = £698.75$ *Ans.* £.698 15s.

2. What is the simple interest of £917 16s. at £5 per cent. per annum, for 7 years?

Ans. £321 4s. 7d.

3. What is the amount of £285 3s. 9d. at £5½ per cent. per annum, from March 5th, 1794, to November 23d, 1794?

Ans. £244 0s. 8½d.

4. If my correspondent is to have £2½ per cent. What will his commission on £785 15s. amount to?

Ans. £19 12s. 10½d.

5. If a broker disposes of a cargo for me, to the amount of £637 10s. on commission at £1¼ per cent. and procures me another cargo of the value of £817 15s. on commission at £1¼ per cent. What will his commission, on both cargoes, amount to?

Ans. £22 5s. 7½d.

G A I S

SIMPLE INTEREST BY DECIMALS. 265

C A S E 2.

The amount, time and ratio given, to find the principal.

RULE.—Multiply the ratio by the time; add unity to the product for a divisor, by which sum divide the amount, and the quotient will be the principal.

E X A M P L E S.

1. What principal will amount to £1045 14s. in 7 years, at £6 per cent. per annum?

Ratio = .06
Multiply by the time = 7

Product = .42

Add 1

Divisor = 1.42 $1045.7(736.4084 \div =$
(£736 8s. 2d.

2. What principal will amount to £3810 in 6 years, at £4½ per cent. per annum? Ans. £3000.

3. What principal will amount to £666 9s. 6d. in 3½ years, at £5½ per cent. per annum? Ans. £563.

C A S E 3.

The amount, principal and time given, to find the ratio.

RULE.—Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

E X A M P L E S.

1. At what rate per cent. will £543 amount to £705 18s. in 5 years?

From the amount = 705.9

Take the principal = 543

Divide by $543 \times 5 = 2715$ $162.90(.66 = \text{£}6 \text{ Ans.}$
162.90

Z

A

2. At what rate per cent. will £391 17s. amount to £449 3s. 14.74qr. in $3\frac{1}{4}$ years? *Ans.* £4 $\frac{1}{4}$.

3. At what rate per cent. will £418 12s. 6d. amount to £546 3s. 8d. in $4\frac{1}{4}$ years? *Ans.* £6 $\frac{3}{4}$.

C A S E 4.

The amount, principal and rate per cent. given, to find the time.

RULE.—Subtract the principal from the amount; divide the remainder by the product of the ratio and principal; and the quotient will be the time.

E X A M P L E S.

1. In what time will £543 amount to £705 18s. at £6 per cent. per annum?

From the amount = 705,9
Take the principal = 543

Divide by $543 \times .06 = 32,58$ 162,9 (5 years, *Ans.*
162,9

2. In what time will £3000 amount to £3810 at $4\frac{1}{4}$ per cent. per annum? *Ans.* 6 years.

To find the Interest of any sum, at 6 per cent. per annum, for any number of Months.

RULE.—If the months be an even number, multiply the principal by half that number: And if the months be uneven, halve the even months, to which annex $\frac{1}{2}$; thus, the half of 19 is 9,5, and multiply the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What

SIMPLE INTEREST BY DECIMALS. 267

4. What is the interest of £345 16s. 6d. for 9 years and 11 months, at 6 per cent. per annum?

Y. M.

9 11

12

2)119 months.

315,825

69,5 = $\frac{1}{2}$ No. of months.

59,5

1729125

3112425

1729125

£205,76875 = £205 15 3 $\frac{1}{4}$ Ans.

Principal = £345 16 6

Amount = £551 11 9 $\frac{1}{4}$

A TABLE of Decimal Parts for every day in the twelfth part of a year, which consists of 365 $\frac{1}{4}$ days.

days	dec. pts.	days	dec. pts.	days	dec. pts.	days	dec. pts.	days	dec. pts.
1	,032	7	,23	13	,427	19	,624	25	,821
2	,066	8	,263	14	,46	20	,657	26	,854
3	,098	9	,296	15	,493	21	,69	27	,887
4	,131	10	,328	16	,526	22	,723	28	,92
5	,164	11	,361	17	,558	23	,756	29	,953
6	,197	12	,394	18	,591	24	,788	30	,986

Note. This table may also be used for the parts of a year, in compound interest, after having worked for a whole year.

To find the Interest of any sum, either for Months, or Months and Days, at 6 per cent. per annum.

RULE.—Multiply the principal by the number of months (and parts, if any, answering to the given number of days in the table) and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be

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be the interest for the given time in shillings and decimal parts of a shilling.

What is the interest of £250 10s. for 19 months and 7 days?

Principal = £250.5

Time = 19,23

7515

5010

22545

2505

Ans. s. 481,7115

= £24 13. 8½d.

Another Method of calculating Interest for Months, at per cent. per annum.

RULE.—If the principal consist of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for one month in shillings and decimal parts: If it consists of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.

E X A M P L E S.

1. What is the interest of £175 for 5 months?

£175 = 17,5 shillings = interest for 1 month.

Mult. by the time = 5

21087,5

Ans. = £4 7s. 6d

2. What is the interest of £255 16s. for months.

Shill.

£255 16 = 25,58 int. for 7 (1 month)

210179,06

£8 19s. 0½d.

SIMPLE

S I M P L E I N T E R E S T, &c. 269

S I M P L E I N T E R E S T I N F E D E R A L M O N E Y.

PROB. 1. *When the principal is given in lawful money, and the interest is required in federal money, at 6 per cent. per annum.*

RULE.—Reduce the shillings, &c. to their equivalent decimal, by inspection, divide the whole by 5, and the quotient is the annual interest : Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.

E X A M P L E S.

1. Required the annual interest of 517*l.* 3*s.* 7½*d.* at 6 per cent. ?

3 <i>s.</i> = ,15	5)517,181	<i>Dols. c. m.</i>
1½ <i>d.</i> = 30	103,436	= 103,43,6
Excess of 12 = 1	Or, 517,181	<i>Ans.</i>
—	2	
,181	—————	<i>Dols. c. m.</i>
	103,4362	= 103,43,6

2. Required the annual interest of 1*l.* in dollars ?

$$\begin{array}{r} 5 \overline{)1,00} \\ 20 \text{ cents; } \textit{Ans.} \end{array}$$

PROB. 2. *When the principal is given in lawful money, and the interest and amount are required in federal money, at 6 per cent.*

RULE.—Reduce the lawful money to federal, then divide the principal by 20, and that quotient by 5 ; add those quotients together, and they are the interest ; or add them to the principal, and their sum is the amount.

E X A M P L E S.

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EXAMPLES.

1. Required the amount of 425*l.* 16*s.* 8*d.* for one year, at 6 per cent ?

$$\begin{array}{r}
 3)425,835 \\
 \underline{34} \\
 20)1419,450 \\
 \underline{5)70,9725} \\
 14,1945 \\
 \underline{\hspace{1.5cm}} \\
 \text{Dols. c. m.} \\
 1504,6170 = 1504,61,7 \text{ Anf.}
 \end{array}$$

2. Required the amount of 112*l.* 4*s.* 6*d.* for 1 year ?

$$\begin{array}{r}
 3)112,225 \\
 \underline{24} \\
 20)374,683 \\
 \underline{5)18,7041} \\
 3,7408 \\
 \underline{\hspace{1.5cm}} \\
 \text{Dols. c. m.} \\
 396,52,79 = 396,52,7,9 \text{ Anf.}
 \end{array}$$

PROB. 3. When the principal is lawful money, and the monthly interest is required in federal money.

RULE.—Reduce the shillings, &c. to decimals, by inspection, then separate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the answer in dollars, cents, &c.

EXAMPLE.

Required the monthly interest of 425*l.* 16*s.* 8*d.* in federal money ?

$$\begin{array}{r}
 3)425,835 \\
 \underline{34} \\
 20)1419,450 \\
 \underline{5)70,9725} \\
 14,1945 \\
 \underline{\hspace{1.5cm}} \\
 \text{Dols. c. m.} \\
 7,69,75 = 7,69,75
 \end{array}$$

PROB.

IN FEDERAL MONEY. 271

PROB. 4. When the principal is federal money, and the interest is required in the same.

RULE.—Work according to the general rule in simple interest, that is, multiply by the rate of interest, separate the two right hand figures of the dollars in the product, and it will give the interest in dollars, cents, &c.

N. B. The figures, which are more than three places to the right hand of the comma, are of no account.

EXAMPLES.

1. What is the annual interest of 537 D. 24c. 6m. at 6 per cent?

D. c. m.

537.24,6

6

32,23476 = 32,234 Ans.

2. What is the interest of 1465 D. 46c. 6m. for 16 months, at 6 per cent. per annum?

D. c. m.

1465.46,6

8 = half the number of months

D. c. m.

117,23728 = 117,237 Ans.

3. What is the interest of 537 D. 34c. 7m. for 19 months, at 6 per cent. per annum?

D. c. m.

537.34,7

9.5 = half the number of months.

2686735

4886122

D. c. m.

51,04765 = 51,047 Ans.

N. B

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N. B. Because there are 4 decimals in the multiplier and multiplier, I cut off 4 figures for them, and two more according to the rule.

PROB. 5. *When the principal is federal money, and the monthly interest is required in the same, at 6 per cent. per annum.*

RULE.—Separate the two right hand figures of the dollars, and you then have the interest of 2 months; half of which is the monthly interest in dollars, cents, &c. If there is but one place, or figure, of dollars, a cypher must be prefixed to the left hand,

EXAMPLES.

1. What is the monthly interest of 9D. 59c. 7m. at 6 per cent. per annum?

$$\begin{array}{r} 2) .09597 \quad \text{c. m.} \\ .047985 = 4,8 \text{ nearly, Ans.} \end{array}$$

2. What is the monthly interest of 100D. 50c. 5m.?

$$\begin{array}{r} 2) 1.00505 \quad \text{c. m.} \\ .50252 = 50,2\frac{1}{2} \text{ Ans.} \end{array}$$

Rules for calculating Interest for Days.

RULE 1.—Multiply the given principal by the given number of days, and that product by the rate on the pound; divide the last product by 365, and it will give the interest.

EXAMPLE.

What is the interest of £360 10s. for 175 days, at 6 per cent.?

$$\frac{360,5 \times 175 \times .06}{365} = £10,37 = £10 7s. 4\frac{1}{2}d. \text{ Ans.}$$

RULE 2.—Multiply the given principal by the given number of days, and divide the product by 6083, for £6 per cent. (the number of days in which

SIMPLE INTEREST BY DECIMALS. 273

any sum will double, at that rate) the quotient will give the answer.

EXAMPLE.

What is the interest of £327 10s. at 6 per cent. per annum, for 210 days?

$$\frac{327.5 \times 210}{6000} = £11,306 = £11 6s. \text{ Ans.}$$

Rule for making a Divisor for any rate per cent.

Multiply 365 by 100, and divide the product by the rate.

$$\text{Thus, for 6 per cent. } \frac{365 \times 100}{6} = 6083 \text{ divisor.}$$

$$\text{For 5 per cent. } \frac{365 \times 100}{5} = 7300 \text{ divisor, \&c.}$$

Hence, when interest is to be calculated on cash accounts, accounts current, or any other accounts, where partial payments are made, or partial debts are contracted; multiply the several balances into the days they are at interest, and the sum of these products, divided as above, will give the interest at £5 or £6 per cent. and for any other rate, make the proper addition or deduction; or find a divisor, as before directed.

EXAMPLE.

On the 1st of January I lent £450 10 6, which I received back in the following partial payments, viz. on the 15th of January, £57 11 9; on the 7th of February, 39 3 10; on the 19th of March, 63 5 2; on the 4th of April, 45; on the 26th of April, 19 12 6; on the 12th of May, 100; on the 10th of June, 60 7 3; and on the first of August, £65 10; What interest is due at 6 per cent.?

Dates

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Dates.			£.	s.	d.	Days.	Products.
January	1	Lent on demand	450	10	6	14	6307
	15	Received in part	57	11	9		
		Balance	392	18	9	24	9430
February	7	Received in part	39	3	10		
		Balance	353	14	11	40	14249
	19	Received in part	63	5	2		
March		Balance	290	9	9	16	4647
	4	Received in part	45	0	0		
		Balance	245	9	9	22	5400
Ditto	26	Received in part	19	12	6		
		Balance	225	17	3	16	3613
	12	Received in part	100	0	0		
May		Balance	125	17	3	29	3650
	10	Received in part	60	7	3		
		Balance	65	10	0	52	3406
August	1	Received in full of the principal	65	10			

6c83)5c6c6 0 5(8 6 4½ Ans.

I have given Peter Truffy a cash credit for £1000, in consequence of which, on the 12th of May, I paid his bill for £250; ditto 27th paid his draught for £280; June 1st he gave me a bill on the Massachusetts bank, at sight, for £290; July the 17th he paid me per receipt £70; August 20th he drew for £750 at sight; ditto 31st he paid me per receipt, £500; September 15th he drew at sight for £135; and 3d of October for £175; October 29th he paid me per receipt, £250; and November 9th £125; November 12th he drew at sight for £375 and ditto 18th for £125; January 1st he paid me per receipt £290, and 20th ditto £210; on the 1st of March

SIMPLE INTEREST BY DECIMALS. 275

March following, he demands a settlement; What then due to me, interest at 6 per cent.?

Dates.			£.	Days.	Products.
May	22	Paid his bill - -	250	13	3750
Ditto	27	Paid his draught -	280		
		Balance	530	5	2650
June	1	Received in part -	290		
		Balance	240	46	11040
July	17	Received in part -	70		
		Balance	170	34	5780
August	20	Paid - - -	750		
		Balance	920	11	10120
Ditto	31	Received in part -	500		
		Balance	420	15	6300
September	15	Paid - - -	135		
		Balance	555	18	9990
October	3	Paid - - -	175		
		Balance	730	26	18980
Ditto	29	Received in part -	250		
		Balance	480	5	2400
November	3	Received in part -	125		
		Balance	855	9	3195
Ditto	12	Paid - - -	375		
		Balance	730	6	4380
Ditto	18	Paid - - -	125		
		Balance	855	62	53010
January	1	Received in part -	290		
		Balance	565	19	10738
Ditto	20	Received in part	210		
		Balance	385	40	15400
March	1				

£165530
then,

276. SIMPLE INTEREST BY DECIMALS.

Then, $73.00) 155130$

$5) 21$ 8 10 Int. at £5 per cent. per annum.
4 5 10 add,

25 14 8 Int. at 6 per cent. on said account,
355 Balance of the account.

6380 14 8 Total balance in my favor.

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days, the first leisure minute; then when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

A owes B the following sums, with the interest on them, at 6 per cent. per annum, as follows, viz. £60 for 7 months; £150 for 5 months; £75 for 9 months; £145 15s. for 7 months, and £397 12s. for 4½ months; What is the amount of principal and interest?

£.	Months.	
60	X 7	= 420
150	X 5	= 750
75.5	X 9	= 679.5
145.75	X 7	= 1020.25
397.6	X 4.5	= 1789.2
		200) 4648.95
		23.24 interest.
		828.85 principal.
		Amount, Ans. £852.09

Not.

D I S C O U N T.

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Note. I divide by 100, the number of months, in which any sum will double at 6 per cent. per annum, and it gives the interest.

When partial payments are made upon notes, bonds, &c. at any interval greater than a year, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

D I S C O U N T

Is an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The present worth of any sum, or debt, due some time hence, is such a sum, as, if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt, then due.

RULE.—As the amount of £100, for the given rate and time, is to £100: So is the given sum or debt, to the present worth.

Subtract the present worth from the given sum, and the remainder will be the discount required.

Example,

As the amount of £100 for the given rate and time, is to the interest of £100 for that time: So is the given sum or debt to the discount required.

E X A M P L E S.

1. What is the discount of £500 at 4 per cent. per annum, due two years hence?

Ans. £40. The present worth of £500 at 4 per cent. per annum, for two years, is £460. The discount is the difference between £500 and £460, which is £40.

BARTER.

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3. What is the present worth of £65 due 15 months hence, at £6 per cent. per annum?

Ans. £58 9 21.

4. What is the discount on £97 10s. due January 22d, this being September 7th, reckoning interest at £5 per cent.?

Ans. £1 15 11.

5. Bought a quantity of goods for £250 ready money, and sold them for £300, payable 9 months hence: What was the gain, in ready money; supposing the discount to be made at £6 per cent.?

Ans. £37 1 73.

BARTER

Is the exchanging of one commodity for another; and teaches traders to proportion their quantities without loss.

CASE I.

When the quantity of one commodity is given; with its value, or that of its integer, that is of 1lb. 1 Cwt. 1 yd. &c. as also the value of the integer of some other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of selling it.

RULE.—Find the value of the given quantity by the concise method, then find what quantity of the other, at the rate proposed, you may have for the same money: Or, if the quantity be given, find from thence the rate of selling it.

1. How much tea, at 9/6 per lb. will be given in barter for 150 gallons of wine, at 12s. 3d. per gallon?

BARTER.

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4. A has broadcloth, at £12 10s. per piece, and B has mace, at 8s. per lb. How many pounds of mace must B give A, for 85 pieces of cloth?

Ans. 1093 $\frac{1}{2}$ lb.

5. A has 7 $\frac{1}{2}$ Cwt. of sugar, at 8d. per lb. for which B gave him 13 $\frac{1}{2}$ Cwt. of flour; What was the flour rated at per lb?

Ans. 4 $\frac{1}{2}$ d.

CASE II.

If the quantities of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

RULE.—Find the separate values of the two given commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quantity and rate may be easily found.

EXAMPLES.

1. Two merchants barter; A has 30 Cwt. of cheese, at 23s. 6d. per Cwt. and B has 9 pieces of broadcloth, at £3 15s. per piece; Which must receive money? And how much?

Ans. B must pay A £1 10s.

2. A and B would barter; A has 150 bushels of wheat at 5s. 9d. per bushel; for which B gives 65 bushels of barley, worth 2s. 10d. per bushel, and the balance in oats, at 2s. 1d. per bushel;—What quantity of oats must A receive from B?

Ans. 325 $\frac{1}{2}$ bushels

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, say, As the ready money price of the one is to its bartering price, so is that of the other, to its bartering price: Next find the quantity required, according to either the bartering or ready money price.

ANS.

EXAMPLES.

EXAMPLE 1.

1. A has ribbands at 2s. per yard ready money; but in barter he will have 2s. 3d. B has broadcloth at 3s. 6d. per yard ready money: At what rate must B value his cloth per yard, to be equivalent to A's bartering price? And how many yards of ribband, at 2s. 3d. per yard, must then be given by A for 488 yards of B's broadcloth?

Ans. B's broadcloth, at £1 16s. 6½d. per yard—7930 yards of ribband.

2. A and B barter; A has 150 gallons of brandy at 7s. 3d. per gallon ready money; but in barter he will have 8s. per gallon: B has linen at 3s. 6d. per yard ready money; How must B sell his linen per yard, in proportion to A's bartering price? And how many yards are equal to A's brandy?

Ans. Barter price is 3s. 10½d. and he must give A 311 yds. 3qr. 3r.

3. P and Q barter. P has linen at 3s. 7d. per yard; but in barter will have 3s. 10d. Q delivers him broadcloth at £1 16s. 6½d. per yard, worth only £1 13s. per yard: Pray, which has the advantage in barter? And how much linen does P give Q, for 148 yards of broadcloth?

Yd. Yds. s.

As 1 : 36/5 :: 148 : 5102 price of the broadcloth. As 3/10 : 1yd. :: 5102s. : 1409½ yds. of linen.—Q has the advantage; for, as 3/7 : 3/10 :: 3;4 : 35/3½ his proportional price.

4. A has 200 yards of linen, at 1s. 6d. ready money per yard, which he barter with B, at 1s. 9d. per yard, taking buttons at 7½d. per gross, which are worth but 6d.—How many gross of buttons will pay

pay for the linen? Who gets the best bargain, and by how much, both in the whole and per cent.?

Yd. d. Yds. d. d. Gross. d. Gross.

As 1 : 31 :: 200 : 4200. As $7\frac{1}{2}$: 1 :: 4200 : 560.

Yd. d. Yds. £.

As 1 : 18 :: 200 : 36 value of A's linen.

Gr. d. Gr. £.

As 1 : 6 :: 560 : 14 value of B's goods. So that B gained £1 of A.

£. £. £. £.

As 14 : 1 :: 100 : 7 or. 10d. per cent.

LOSS and GAIN

Is an excellent Rule, by which merchants and traders discover their profit or loss, per cent. or by the gross : It also instructs them to raise or fall the price of their goods, in as to gain or lose so much per cent. &c.

CASE I.

To know what is gained or lost per cent.

RULE.—First see what the gain or loss is, by Subtraction; then, As the price it cost is to the gain or loss; so is £100 to the gain or loss per cent.

EXAMPLES.

1. If I buy serge at 5s. per yard, and sell it again, at 5s. 8d. per yard; What do I gain per cent. or in laying out £100?

Sold for $\begin{matrix} s. & d. \\ 5 & 8 \end{matrix}$ [See process next page.]
Cost 5 0

8d gain per yard.

N. B. The first questions in the several Cases, serve to elucidate each other.

LOSS AND GAIN.

As 60 : 8 :: 100 : 13 1/3
 60 800 (13 1/3)
 60
 200
 180

2. If I buy serge at 5/8 per yard, and sell it again at 5s. per yard; what do I lose per cent. or in laying out £100?

s. d.	d.	d.	£.	s. d.	
Cost 5 8	As 68 : 8 :: 100 : 11 15 3/4		£.	11 15 3/4	60) 400 (6 360
Sold for 5 0					40
					12
					60) 48 (8 480

8d. Loss

Ans. £13 6/8

3. If I buy a Cwt. of tobacco for £9 5/8, and sell it again at 1s. 10d. per lb., do I gain or lose? And what per cent.?

£.	s.	d.
Sold for 10 5 4		
Cost 9 5 8		
£. s. d. £0 18 8 gained in the gross.		
[ad. 1/11] 11 4 value at 1s. per lb.		
0 18 8 value at 2d. per lb.		
10 5 4 value at 1s. 10d. per lb.		
£. s. d.	s. d.	£.
As 9 6 8 : 18 8 :: 100 : 10		Ans. £10 per cent.

4. A draper bought 60 yards of cloth at 28s. per yard, and 38 yards of ditto, at 14s. per yard, and sold them one with another, at 26s. per yard: Did he gain or lose? And what per cent?

$$\begin{array}{l} yd. \quad s. \quad \pounds. \quad yds. \quad yd. \quad s. \quad yds. \quad \pounds. \quad s. \\ As \ 1 : 28 :: 60 : 84 \quad As \ 1 : 26 :: 98 : 127 \ 8 \end{array}$$

$$\begin{array}{l} yd. \quad s. \quad yds. \quad \pounds. \quad s. \quad \pounds. \quad s. \\ As \ 1 : 14 :: 38 : 26 \ 14 \quad \text{Sold for } 127 \ 8 \end{array}$$

Prime cost 110 12

Whole prime cost $\pounds$ 110 12

Gain in the gross 16 16

$$\begin{array}{l} \pounds. \quad s. \quad \pounds. \quad s. \quad \pounds. \quad \pounds. \quad s. \quad d. \\ As \ 110 \ 12 : 16 \ 16 :: 100 : 15 \ 3 \ 9\frac{1}{2} \text{ per cent.} \end{array}$$

5. Bought sugar at 6½d. per lb, and sold it at £2 3s. 9d. per Cwt. What was the gain or loss per cent?

$$\begin{array}{l} \text{lb.} \quad \text{lb.} \\ As \ 1 : 6\frac{1}{2}d :: 112 : \pounds 3 \ 0s. \ 8d. \end{array}$$

Prime cost $\pounds$ 3 0 8 per Cwt. $\pounds$ 3 0 8 : 16 11 :: 100 : 17 17 8½

Sold at 2 3 9 per Cwt. (Loss per cent, Ans.)

Loss 16 11 in the whole.

6. At 2½d. in the shilling profit; how much per cent?

$$\begin{array}{l} s. \quad d. \quad \pounds. \quad \pounds. \quad s. \quad d. \\ As \ 1 : 2\frac{1}{2} :: 100 : 20 \ 16 \ 8 \text{ Ans.} \end{array}$$

7. At 4s. 6d. in the pound profit; how much per cent?

$$\begin{array}{l} \pounds. \quad s. \quad d. \quad \pounds. \quad \pounds. \quad s. \quad d. \\ As \ 1 : 4 \ 6 :: 100 : 22 \ 10 \text{ Ans.} \end{array}$$

8. If I buy candles, at 1s. 6d. per lb, and sell them again, at 2s. per lb, and allow 3 months for payment: What do I gain per cent?

$$\begin{array}{l} d. \quad d. \quad \pounds. \quad \pounds. \quad s. \quad d. \\ As \ 18 : 24 :: 100 : 133 \ 6 \ 8 ; \text{ then, by Disc.} \end{array}$$

$$\begin{array}{l} Mo. \quad \pounds. \quad Mo. \quad \pounds. \quad s. \\ As \ 12 : 6 :: 3 : 1 \ 10 \end{array}$$

Then, As 101 10 : 1 10 :: 133 6 8 : 1 19 4½, which taken from $\pounds$ 133 6 8d. leaves $\pounds$ 131 7 3½d. therefore, Ans. $\pounds$ 31 7 3½d.

9. If

9. If I buy cloth at 13s. per yard, on 8 months credit, and sell it again at 12s. ready money, do I gain or lose? And what per cent.?

Mo. £. Mo. £. As 12 : 0 :: 8 : 4. As 104 : 13 :: 100 : 12 6
So that 13s. on 8 months credit, at 6 per cent. is equal to 12s. 6d. ready money; then,

Prime cost 12 6 ready money } As 12 6 : 6 :: 100 : 4
Sold at 12 0 ready money }

Lost 0 6 in the yard. } Ans. lost £ 4.

In casting up the amount of goods bought, imported or exported; to the prime cost of such goods we must add all the charges upon them, in order to find the price they stand us in.

10. Suppose I import from France 12 bales of cloth, each bale containing 10 pieces, which, with the charges there, amounted to £108; I pay duty here 5s. 6d. per piece, for freight £3 10s. and the portorage 7s. 6d.; What does it stand me in per piece? And how must I sell it per piece to gain £10 per cent.?

12 bales. First cost 108 0 0
Each 10 pieces. Duty 33 0 0
Freight 3 10 0
In all 120 pieces, at 5s. 6d. Portorage 0 7 6

Whole cost £144 17 6
Pieces. £. s. d. pieces.
As 120 : 144 17 6 :: 1

As 10 : 0 : 10 :: 1 : 4 12
(12) 144 17 6 di vider, 20 1 : 4 12 cost per piece.
(10) 12 1 5 1/2 } 2 5 gain per piece }
at 10 per cent. }
£ 6. 6 1/2

Ans. } Prime cost per piece, £1 4 12
And I must sell it at £3 5 6 1/2 per piece

LOSS AND GAIN.

281

CASE II.

To know how a commodity must be sold, to gain or lose so much per cent.

RULE.—As £100 is to the price; so is £100 with the profit added or loss subtracted, to the gaining or losing price.

EXAMPLE 1.

1. If I buy a quantity of serge, at 5s. per yard; how must I sell it per yard to gain £13 bs. 8d. per cent.?

As 100 :: 5 s. :: 113 6 8
 Or thus, £. s. d.
 100 20 113 6 8

2000 2266
 13 12

24000 27200
 5

24,000 185,000(5
 120

16

12

24,000 192,000(8
 192

Add 8 = $\frac{1}{2}$ of 12 = 13 4d.

2. 8 00

Ans. 5 8 per yard

2. If a barrel of powder cost £4, how must it be sold to lose 10 per cent.?

$\text{£. } \text{£. } \text{£. } \text{£. s. d.}$
As 100 : 4 :: 90 : 3 12 *Ans.*

Or thus,

93

4

 $\text{£} 3, 60$

20

3. Bought cloth, at 15s. per yard, which not proving so good as I expected, I am content to lose 15 $\frac{1}{4}$ per cent. by it: How must I sell it per yard?

$\text{£. } \text{s. } \text{£. } \text{s. d.}$
As 100 : 15 :: 82 $\frac{1}{2}$: 12 4 $\frac{1}{2}$ *Ans.* 12s. 4 $\frac{1}{2}$ d.

4. If 120lb of steel cost 7 $\frac{1}{2}$ l; how must I sell it per lb to gain 15 $\frac{1}{4}$ per cent.?

$\text{lb. } \text{£. } \text{lb. } \text{s. d. } \text{£. } \text{s. d. } \text{£. } \text{s. d.}$
As 120 : 7 :: 1 : 2 2 *As* 100 : 115 $\frac{1}{2}$: 14
 (per lb. *Ans.*)

5. Bought fish in Newburyport, at 10s. per quintal, and sold it at Philadelphia at 17s. 6d. per quintal; now, allowing the charges to be 2s. 6d. per quintal, and considering I must lose 20l. per cent. by remitting my money home; What do I gain per cent.?

Selling price 17 6 Philadelphia currency per quintal.
 Charges = 2 6 Ditto.

15 0 Ditto.

$\text{£. } \text{s. } \text{£. } \text{s.}$
As 100 : 15 :: 80 : 12 Newengland currency.

Sold at 12s. per quintal.

Bought at 10s. per quintal.

Gained 2s. per quintal— $\text{£. } \text{s. } \text{£. } \text{s.}$
As 10 : 2 :: 100 : 20 per cent. gained, *Ans.*

6. Bought 10 gallons of brandy, at 21s. per gallon, but, by accident, 10 gallons leaked out.

what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of £10 per cent. ?

Gal. £. Gal. s.
 50 gall. at 4s. per gall. = £10 As 40 : 10 :: 1 : 5
 10 gallons leaked out. £. s. £. s. d.
 — As 100 : 5 :: 110 : 5 6 Ans.
 40 gallons remain.

CASE III.

When there is gained or lost per cent. To know what the commodity cost.

RULE.—As £100 with the gain per cent. added, or loss per cent. subtracted, is to the price; so is £100 to the prime cost.

EXAMPLES.

1. If 1 yard of cloth be sold, at 5s. 8d. and there is gained £13 6 8 per cent. what did the yard cost ?

£.	s.	d.	s.	d.	£.
As	113	6	8	5	8 :: 100
	20		12		68
	2266		68		800
	12				600
	27200		272,00	68	100
			20		

272,00)1360100(5s. prime cost, Ans.
 1360

2. If 12 yards of cloth are sold at 15s. per yard, and there is £7 10s. loss per cent. in the sale; what is the prime cost of the whole ?

Yd.	s.	Yds.	£.	£.	s.	£.	£.	s.	d.
As	1	15	12	9	As	92	10	9	100 : 9 13 6
B b									

(Ans.
 3. If

3. If $19\frac{1}{2}$ Cwt. sugar be sold, at 4*l.* 5*s.* per Cwt. and I gain 15*l.* per cent. What did it cost per Cwt.?

$$\begin{array}{ccccccc} \text{£.} & \text{£. s.} & \text{£.} & \text{£. s. d.} & & & \\ \text{As } 115 : 45 :: 100 : 81310\frac{1}{4} - \text{Ans.} \end{array}$$

CASE IV.

If by wares sold at such a rate, there is so much gained or lost per cent. To know what would be gained or lost per cent. if sold at another rate.

RULE.—As the first price is to £100 with the profit per cent. added, or loss per cent. subtracted; so is the other price, to the gain or loss per cent. at the other rate.

N. B. If your answer exceed £100 the excess is your gain per cent. but if it be less than £100, that deficiency is the loss per cent.

EXAMPLES.

1. If cloth, sold at 5*s.* 8*d.* per yard, be £13 6*s.* 8*d.* profit per cent. What gain or loss per cent. shall I have, if I sell the same at 5*s.* per yard?

$$\begin{array}{ccccccc} \text{s. d.} & \text{£. s. d. s.} & & & & & \\ \text{As } 58 : 11368 :: 5 \end{array}$$

$$\begin{array}{r} 12 \quad 20 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \quad 2266 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \hline \end{array}$$

$$\begin{array}{r} 27200 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 68136000(300,0 \quad 100 - 100 = 0 \text{ Ans. } I \text{ neither gain} \\ \hline \end{array}$$

$$\begin{array}{r} 136 \\ \hline \end{array}$$

$$\begin{array}{r} \text{£ } 100 \\ \hline \end{array}$$

$$\begin{array}{r} ,000 \\ \hline \end{array}$$

2. If

EQUATION OF PAYMENTS. 291

2. If cloth, sold at 4s. per yard, be £10 per cent. profit; what shall I gain or lose per cent. if sold at 3s. 6d. per yard?

s. £. s. d. £.
As 4 : 110 :: 3 6 : 96½ £. £. £.
Then, $100 - 96\frac{1}{2} = 3\frac{1}{2}$ per cent. loss,
(Ans.)

3. I sold a watch for £50, and by so doing, lost £17 per cent. whereas I ought, in trading, to have cleared £20 per cent. How much was it sold under its real value?

£. £. £. £. s. d. £. £. s. d.
As 89 : 50 :: 100 : 60 4 9½ As 100 : 60 4 9½ ::
£. £. s. d. £. s. d. £. £. s. d.
120 : 72 5 9½ then, $72\ 5\ 9\frac{1}{2} - 50 = 22\ 5\ 9\frac{1}{2}$ Ans.

EQUATION OF PAYMENTS

Is the finding of a time to pay at once, several debts due at different times, so that no loss shall be sustained by either party.

RULE 1.—Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time, or that required.

EXAMPLES.

1. A owes B £380 to be paid as follows, viz. £100 in 6 months, £120 in 7 months, and £160 in 10 months: What is the equated time for the payment of the whole debt?

* This rule is founded upon a supposition, that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms.

EQUATION OF PAYMENTS.

$$100 \times 6 = 600$$

$$120 \times 7 = 840$$

$$160 \times 10 = 1600$$

$$100 + 120 + 160 = 380 \quad 3040 \text{ (8 months, Ans.)}$$

$$3040$$

2. A owes B £104 15s. to be paid in $4\frac{1}{2}$ months, £161 to be paid in $3\frac{1}{2}$ months, and £152 5s. to be paid in 5 months; What is the equated time for the payment of the whole? *Ans. 4 months and 8 days.*

3. There is owing to a merchant £698 to be paid £178 ready money, £200 at 3 months, and £320 in 8 months; I demand the indifferent time for the payment of the whole? *Ans. $4\frac{1}{2}$ months.*

4. The sum of £49 1s. 6d. is to be paid, $\frac{1}{2}$ at 6 months, $\frac{2}{3}$ at 8 months, and $\frac{1}{6}$ at 12 months; What is the mean time for the payment of the whole.

Ans. $7\frac{2}{3}$ months.

RULE 2.—See by Rule 1st, at what time the first man, mentioned, ought to pay in his whole money; then, as his money is to its time, so is the other's money to his time inversely, which, when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the true time of the second's payment.

EXAMPLES.

A is indebted to B £150, to be paid £50 at 4 months, and £100 at eight months: B owes A £250 to be paid at 10 months: It is agreed between them, that A shall make present pay of his whole debt, and that B shall pay his so much the sooner, as to balance that favor; I demand the time at which B must pay the £250, reckoning simple interest?

COMMISSION or FACTORAGE.

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$$50 \times 4 = 200$$

$$100 \times 8 = 800$$

$$50 + 100 = 150 \quad 100 \div 150 = \frac{2}{3} \text{ months, A's equated time.}$$

$$90$$

$$10$$

$$10$$

$$10$$

$$f. \quad mo. \quad f. \quad mo. \quad mo. \quad mo. \quad mo.$$

As 150 : 6 $\frac{2}{3}$:: 250 : 4 then 10 $\div$ 4 = 2 $\frac{1}{2}$ time of B's (payment.

2. A merchant has 1200*l.* due to him, to be paid $\frac{1}{3}$ at 2 months, $\frac{1}{3}$ at 3 months, and the rest at 6 months ; but the debtor agrees to pay $\frac{1}{2}$ down : How long may the debtor detain the other half, so that neither party may sustain loss ?

$$mo. \quad mo.$$

$$\frac{1}{3} \times 2 = 0\frac{2}{3}$$

$$\frac{1}{3} \times 3 = 1$$

$$\frac{1}{3} \times 6 = 2$$

$$\text{Equated time } 4\frac{1}{2}$$

Now, as $\frac{1}{2}$ was paid 4 $\frac{1}{2}$ months before it was due, it is reasonable that he should detain the other $\frac{1}{2}$ 4 $\frac{1}{2}$ months after it became due, which, added, gives 8 $\frac{1}{2}$ months, the true time for the second payment.

COMMISSION or FACTORAGE.

Is an allowance of so much per cent. to a factor, or correspondent abroad, for buying and selling goods, for his employer.

EXAMPLES.

1. What comes the commission on 539*l.* 12*s.* 6*d.*, at 4 $\frac{1}{2}$ per cent. ?

539

* The method of working questions in this Rule, Brokerage and the first case of Insurance, is the same as in Simple Interest.

b. b.

£. s. d.
539 12 9
4½

2158 11 0
Product of 4 = 269 16 4½

£2428 7 4½

20

s. 5167

12

d. 8108

2. My factor receives 1008*l.* to lay out after having deducted his commission of 5*l.* per cent.—What does his commission amount to?

Here, as his commission is to be deducted from the given sum, it is evident that I ought not to pay him commission on his own money (which, however, is often unjustly practised) therefore,

£. £. £. £.
As 105 : 5 :: 1008 : 48 4½.

3. My correspondent writes me that he has purchased goods to the value of 673*l.* 12*s.*—What does his commission on that sum amount to, at 3½ per cent.?

Ans. £23 12 6

BROKERAGE

Is an allowance of so much per cent. to a person called a Broker, for assisting merchants or factors in purchasing or selling goods.

EXAMPLES.

1. What is the brokerage upon 525*l.* 10*s.* at 5*l.* or ½ per cent.?

BUYING and SELLING STOCKS.

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$$\begin{array}{r|l} \text{5s.} & \frac{1}{4} \\ \hline \text{£. s.} & 525 \ 10 \end{array}$$

$$\begin{array}{r} 1,81 \ 7 \ 6 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 6,27 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 3,30 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 1,20 \end{array}$$

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$$\begin{array}{r} 1,20 \end{array}$$

$$\begin{array}{r} 1,20 \end{array}$$

5 shillings being an aliquot part of 14, divide by that part, and the quotient is the answer.

$$\text{Ans. } £ \ 1 \ 6 \ 3\frac{1}{2}$$

2. What is the brokerage upon 673*l.* 16*s.* at $\frac{3}{4}$ per cent. ?

$$\text{Ans. } £ \ 4 \ 4 \ 2\frac{1}{2}$$

3. If a broker sell goods to the amount of 709*l.* 19*s.* 11*d.*—What is his demand, at $1\frac{1}{2}$ per cent. ?

$$\text{Ans. } £ \ 10 \ 12 \ 11\frac{1}{2}$$

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies, banks, &c. and the buying and selling certain sums of money in those funds, is not unusual.

EXAMPLES.

1. What is the purchase of 275*l.* 15*s.* bank stock, at 74*l.* per cent. ?

$$\begin{array}{r|l} \text{74} \frac{1}{2} \text{ per cent.} & \text{of } 100 \text{ } \\ \hline \text{therefore, take parts for the deficiency, and subtract the sum of those parts from the given sum.} & \end{array}$$

$$\begin{array}{r|l} \text{£ } 20 & \frac{1}{2} \\ \hline \text{£ } 5 & \frac{1}{4} \\ \hline \text{£ } \frac{1}{2} & \frac{1}{16} \end{array}$$

$$\begin{array}{r} 275 \ 15 \ 0 \\ \hline 55 \ 3 \ 0 \\ \hline 13 \ 15 \ 9 \\ \hline 1 \ 7 \ 6\frac{1}{2} \end{array}$$

$$\text{Subtract } 70 \ 6 \ 3\frac{1}{2}$$

$$\text{Ans. } £ \ 105 \ 8 \ 3\frac{1}{2}$$

Or, If I had taken parts for $74\frac{1}{2}\%$ the rate per cent. then the sum of the several quotients would have been the answer as above.

When the price is above 100, take parts for the surplus of the price above 100, and add them to the given sum for the answer.

2. What is the purchase of 1029*l.* 10*s.* 6*d.* bank stock, at $110\frac{1}{4}\%$ per cent. ?

<i>£</i>	<i>s.</i>	<i>d.</i>	
10	10	6	at 100 per cent.
$\frac{1}{4}$	19	0	at 10 per cent.
	2	11	at $\frac{1}{4}$ per cent.

Ans. *£* 1135 1 0 at $110\frac{1}{4}\%$ per cent.

3 What does 1600*l.* capital stock in the Massachusetts Bank, come to, at $120\frac{1}{4}\%$ per cent. ?

Ans. *£* 1922.

POLICIES OF INSURANCE.

Insurance is a security, or assurance, by means of a Writ called a Policy, to indemnify the insured of such losses as shall be specified in the policy subscribed by the insurer or insurers, by which the underwriters oblige themselves to make good and effectual the property insured, in consideration of a certain premium at a stipulated rate per cent. (which varies according to the risque) to be immediately paid down, or otherwise secured according to the tenor of the agreement.

The average loss is 10 per cent.—that is, if the insured suffer any damage or loss, not exceeding 10 per cent. he bears it himself, and the insurers are free.

A policy should be taken out for a sum sufficient to cover the principal and premium, and the business of this rule is, in general, to calculate what that sum should be.

CASE I.

When the premium, at a certain rate per cent. for insuring a sum, is required, the operation is the same as in interest, or commission.

1. What is the premium upon 537*l.* 15 *s.* 9 at 6½ per cent. ?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 537 \quad 15 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3226 \quad 14 \quad 6 \\ \frac{1}{2} = 268 \quad 17 \quad 10\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 34|95 \quad 12 \quad 4\frac{1}{2} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 19|12 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 1|48 \\ 4 \\ \hline \end{array}$$

$$1|94 \quad \text{Ans. } \text{£} 34 \text{ } 19 \text{ } 4\frac{1}{2}.$$

CASE II.

To find the sum for which a policy should be taken out to cover a given sum.

RULE.—Take the premium from 100*l.* and say, as the remainder is to 100 ; so is the sum adventured to the policy.*

1. It

* Now, it is plain, that, if I want to recover 9*l.* I must in this case, insure upon 100*l.* therefore, to recover 759*l.* I must insure upon 825*l.* for when 8 per cent. for premium, is deducted, I shall have 759*l.* remaining neat.

For 825*l.* = Sum insured upon at 8 per cent.

66 = Premium to be deducted,

£. 759 = The first outset,

1. It is required to cover £759, premium 8 per cent.—For what sum must the policy be taken?

$$\begin{array}{r} 100 \\ 8 \end{array}$$

$$92 : 100 :: 759$$

$$100$$

$$92)75900(\text{£}825 \text{ Ans.}$$

$$736$$

$$230$$

$$184$$

$$460$$

$$460$$

2. A merchant sent a vessel and cargo to sea, valued at £1525; What sum must the policy be taken out for, to cover his property, premium $19\frac{1}{2}$ per cent.?

$$100$$

$$19,5$$

$$80,5 : 100 :: 1525 : \text{£}1894 \text{ } 8 \text{ } 2\frac{1}{2} \text{ Ans.}$$

CASE III.

When a policy is taken out for a certain sum in order to cover a given sum: To find the premium, say; As the policy is to the covered sum: so is 100 to a fourth number, which, being taken from 100, will leave the premium.

If a policy be taken out for £1250 to cover £500, What is the premium per cent.?

$$1250 : 500 :: 100$$

$$100$$

$$1250)50000(40 \text{ and } \text{£}100 - 40 = \text{£}60 \text{ Ans.}$$

CASE

COMPOUND INTEREST. 193

C A S E IV.

When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

RULE.—Deduct the premium per cent. from 100, and say, As 100 is to the remainder; so is the policy to the sum required to be covered;

If a policy be taken out for £1250 at 60 per cent.—What is the adventure, or sum to be covered?

100

60

100 : 40 :: 1250

40

100)50000 (£500 Ans.

COMPOUND INTEREST.

Compound Interest is that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becomes a part of the principal, at the end of each stated time of payment.

GENERAL RULE.—Find the amount of the given principal, for the time of the first payment, by simple interest; next, find the interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last amount as the principal for the next payment. The given principal being subtracted from the last amount, the remainder will be the compound interest.

E X A M P L E S.

1. What will £480 amount to in 8 years, at 6 per cent. per annum?

Principal

300 COMPOUND INTEREST.

Principal, 480
Rate of int. 6

£28|80

20

s. 16|00

Prin. 1st year, 480

Int. of ditto, 28 16

Prin. 2d year, 508 16

6

£30|52 16

20

s. 10|56

12

d. 6|72

4

qrs. 2|88

Prin. 2d year, 508 16 0

Int. of ditto, 30 10 6½

Prin. of 3d. yr. 539 6 6½

6

£32|35 19 3

20

s. 7|19

12

d. 2|31

4

gr. 1|24

Prin. 3d year. £539 6 6½

Int. of ditto, 32 7 2½

Amount, Ans. £571 13 8½

Sub. 1st prin. 480

Compound int. } 91 13 8½
for 3 years.

2. What is the compound interest of £740 for 6 years, at £4 per cent. per annum?

Ans. £196 6 8½

3. What will £150 amount to in a year, at £2 per cent. per month?

Ans. £190 4 6

When

COMPOUND INTEREST. 301

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner find the amount for every succeeding year.

When the rate is at 6 per cent. per annum, you may

1. Divide the principal by 20, and that quotient by 5: These quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

EXAMPLES.

What is the amount of £480, at 6 per cent. per annum, for 4 years? Of the same sum at 5 per cent. per annum, for 4 years?

20)480	20)480
5) 24	24
4 16	
10)508 16 amt. of the 1st yr.	10)504 amount of 1st yr.
5) 25 8 9½	25 4
5 1 9	
20)539 6 6½ ditto of 2d.	20)539 4 = ditto of 2d.
5) 26 19 8½	26 9 2½
5 7 10½	
20)571 13 8½ do. of 3d.	20)555 13 2½ = do. 3d.
5) 28 11 8	27 16 7½
5 14 4	
£605 19 8½ ditto of 4th.	£583 8 10 = do. of 4th. Ans.
(Ans.	

COMPOUND

C c

302 COMPOUND INTEREST by DECIMALS.

COMPOUND INTEREST by DECIMALS.

A TABLE of the Amount of 1*l.* at 6 per Cent. per Annum, for Months.

Mo.	£.dec. part.	Mo.	£.dec. part.	Mo.	£.dec. parts.
1	1,00487	5	1,02457	9	1,04467
2	1,00976	6	1,02956	10	1,04975
3	1,01467	7	1,03457	11	1,05486
4	1,01961	8	1,03961	12	1,06

A TABLE of the Amount of 1*l.* from 1 day to 31 days, at 6 per Cent. per Annum.

days	£.dec. part.	days	£.dec. part.	days	£.dec. parts.
1	1,00010	12	1,00197	22	1,00361
2	1,00032	13	1,00213	23	1,00378
3	1,00049	14	1,00230	24	1,00394
4	1,00065	15	1,00246	25	1,00410
5	1,00082	16	1,00263	26	1,00427
6	1,00098	17	1,00279	27	1,00443
7	1,00115	18	1,00295	28	1,00460
8	1,00131	19	1,00312	29	1,00476
9	1,00147	20	1,00328	30	1,00493
10	1,00164	21	1,00345	31	1,00509
11	1,00180				

A TABLE of the Amount of 1*l.* at $\frac{1}{2}$ per Cent. per Month, as practised at the Massachusetts Bank.

Mon.	£.dec. part.	Mon.	£.dec. part.	M.	£.dec. parts.
1	1,005	5	1,025	9	1,045
2	1,01	6	1,03	10	1,05
3	1,015	7	1,035	11	1,055
4	1,02	8	1,04	12	1,06

CASE I.

When the principal, the rate of interest, and time, are given, to find either the amount or interest.

RULE 1.—Find the amount of £1 for one year, at the given rate per cent.

1. Involve

COMPOUND INTEREST BY DECIMALS. 303

2. Involve the amount, thus found, to such power, as is denoted by the number of years—or, in Table 1st, at the end of Annuities, under the rate, and against the given number of years, you will find the power.*

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the interest.

EXAMPLES.

1. What is the compound interest of £600 for 4 years, at 6 per cent. per annum?

$$\text{Multiply by } 1,06 = \begin{cases} \text{Amount of } \text{£}1 \text{ for 1 year,} \\ \text{at 6 per cent. per annum.} \end{cases}$$

$$1,1236 = 2d \text{ power.}$$

$$\text{Multiply by } 1,1236$$

$$1,16247696 = 4th \text{ power.}$$

$$\text{Multiply by } 600 = \text{principal.}$$

$$757,48617600 = \text{amount.}$$

$$\text{Subt. } 600$$

$$157,486176 = \text{£}157 \text{ } 9s. \text{ } 8\frac{1}{2}d. = \text{Interest required.}$$

By

* The amounts of 1*l.* in this Table, are so many powers of the amount of 1*l.* for 1 year, whose indices are denoted by the number of years.

Note. When the given time consists of years and months, or years, months and days; first seek the amount of 1*l.* in the Table for years, then, in the Table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

Thus, if the amount of 180*l.* in $5\frac{1}{2}$ years, at 6 per cent. per annum, were required; the amount of 1*l.* for 5 years = 1,33822*l.* ditto for 6 months = 1,03956*l.* Now, $1,33822 \times 1,03956 \times 180 = \text{£}661,2341$. *Ans.*

304 COMPOUND INTEREST *by* DECIMALS.

By TABLE I.

Tabular amount of £1 for 4 years, } = 1,262,4769
 at 6 per cent. per annum, }
 Multiply by the principal = 600

Amount = 757,486,1400

Another way of working Compound Interest for years, months, and days, which is much more concise than the preceding method.

RULE.—To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of years, months and days, found in the following Tables, and their sum will be the logarithm of the amount for the given time, which being found in any Table of logarithms, the natural number corresponding thereto, will be the answer.

Logarithmic TABLES, at 6 per Cent. per Annum, for Years, Months, and Days.

Ys.	dec. pts.	Y.	dec. pts.	Y.	dec. pts.	Y.	dec. pts.	M.	dec. pts.
1	,025306	11	,278366	21	,531426	31	,784586	1	,00166
2	,050612	12	,303672	22	,556732	32	,809792	2	,004321
3	,075918	13	,328978	23	,582038	33	,835098	3	,006466
4	,101224	14	,354284	24	,607344	34	,860404	4	,0086
5	,12653	15	,37969	25	,63265	35	,88571	5	,010724
6	,151896	16	,404996	26	,657956	36	,911016	6	,012837
7	,177142	17	,430202	27	,683262	37	,936322	7	,01494
8	,202448	18	,455508	28	,708568	38	,961628	8	,017033
9	,227754	19	,480814	29	,733974	39	,986934	9	,019116
10	,25306	20	,50612	30	,75928	40	,1,01224	10	,021289
								11	,023252
days		D		D		D		D	
1	,000071	8	,000571	14	,000999	20	,001426	26	,001852
2	,000143	9	,000642	15	,00107	21	,001497	27	,001923
3	,000215	10	,000713	16	,001142	22	,001568	28	,001994
4	,000287	11	,000785	17	,001213	23	,001639	29	,002065
5	,000358	12	,000857	18	,001284	24	,00171	30	,002136
6	,000429	13	,000928	19	,001355	25	,001781	31	,002207
7	,0005								

What

COMPOUND INTEREST *by* DECIMALS. 305

What is the amount of £132 10s. at 6 per cent. per annum, for 9 years, 8 months, and 15 days?

$$\begin{array}{r} \text{To the Log. of } £132,5 = 2,12322 \\ \text{Add } \left\{ \begin{array}{l} \text{Log. for 9 years} = ,22775 \\ \text{ditto for 8 months} = ,01703 \\ \text{ditto for 15 days} = ,00107 \end{array} \right. \end{array}$$

2,36807 the

nearest to which, in the Table of Logarithms, is 2,36810, and the natural number answering thereto is 233,4 = £233 8s. *Ans.*

C A S E II.

When the amount, rate and time are given, to find the principal.

RULE.—Divide the amount by the amount of £1 for the given time, and the quotient will be the principal.

Or, If you multiply the present value of 1l. for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or present worth.

E X A M P L E.

1. What is the present worth of £757 9s. 8½d. due 4 years hence, discounting at the rate of £6 per cent. per annum?

By Tab. 1st, divide by the tab. }
 ular amount of 1l. for 4 years, } = 1,2624769)757,4861400(600L.
(*Ans.*)

By TABLE II.

$$\begin{array}{r} \text{Amount} = 757,48614 \\ \text{Mult. by the present worth of } £1 \text{ for } \left. \begin{array}{l} 4 \text{ years, at 6 per cent. per annum.} \end{array} \right\} = ,7920936 \\ \hline \text{Ans. } 599,999923582704 + = £600 \end{array}$$

306 COMPOUND INTEREST *by* DECIMALS.

CASE III.

When the principal, rate and amount are given, to find the time.

RULE.—Divide the amount by the principal ; then divide this quotient by the amount of £ 1 for 1 year, this quotient by the same till nothing remain, and the number of the divisions will shew the time.

Or, Divide the amount by the principal, and the quotient will be the amount of £ 1 for the given time, which seek under the given rate in Table 1st, and in a line with it, you will see the time.

E X A M P L E.

1. In what time will £ 600 amount to £ 757 9s. 8½d. at 6 per cent. per annum compound interest ?

Divide the amount } = 600)757,486,76(1,26247696
by the principal, } (= quotient to be divided
by 1,06 till it can be had no more ; or you may find
it in Table 1st, under 6 per cent. and against 4 years.
Divide by 1,06)1,26247696

1,06)1,191016

Four divisions shew the
time to be 4 years.

1,06)1,1236

1,06)1,06

CASE IV.

When the principal, amount and time are given, to find the rate per cent.

RULE.—Divide the amount by the principal, and the quotient will be the amount of £ 1 for the given time ; then, extract such root as the time denotes, and that root will be the amount of £ 1 for 1 year,
from

DISCOUNT by COMPOUND INTEREST. 307

from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of £1 for the time, as above directed, look for it in Table 1st, even with the given time, and directly over the amount you will find the ratio.

EXAMPLE.

At what rate per cent. per annum will £600 amount to £757 $9/8\frac{1}{2}$ in 4 years?

600)757.48614(1.2624769.—Now, the time being 4 years, the 4th root of this quotient minus 1 will be the ratio.

1.26247690(1.123599 + and 1.123599(1.05599+, and 1.05599—r = .06 *Ans.*

DISCOUNT by COMPOUND INTEREST.

CASE I.

The sum, or debt to be discounted, the time and rate given, to find the present worth.

RULE.—Divide the debt by that power of the amount of £1 for 1 year, denoted by the time, and the quotient will be the present worth, which, subtracted from the debt, will leave the discount.

EXAMPLES.

1. What is the present worth, and discount, of £600, due 3 years hence, at £6 per cent. per annum, compound interest?

Divide by $1.06|^3 = 1.19101$ 600,00000(503.7741 = £503 $15/5\frac{1}{2}$ present worth; and £600—£503 $15/5\frac{1}{2}$ = £96 4s. 6 $\frac{1}{2}$ d. = Discount.

By

308 ANNUITIES or PENSIONS in ARREARS.

By TABLE 2d. In this table, corresponding to the time and rate, we

have $839619 =$ present worth of $\pounds 1$ for the time and rate.

Mult. by 600 = debt, or principal.

$503,771400 =$ present worth of the debt.

2. What is the present worth of $\pounds 312$ 10s. due 2 years hence, at $4\frac{1}{2}$ per cent. per annum compound interest? *Ans.* $\pounds 286$ 3s. 3d. 2,97 qrs.

3. What ready money will discharge a debt of $\pounds 1000$ due 4 years hence, at $\pounds 5$ per cent. per annum compound interest? *Ans.* $\pounds 822$ 14s. Od. 2,304 qrs.

ANNUITIES or PENSIONS in ARREARS, at COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent. are given, to find the amount.

RULE 1.—Make 1 the first term of a Geometrical Progression, and the amount of $\pounds 1$ for 1 year at the given rate per cent. the ratio.

2. Carry the series to so many terms as the number of years, and find its sum.

3. Multiply the sum, thus found, by the given annuity, and the product will be the amount sought. Or. Multiply the amount of $\pounds 1$ for 1 year into itself so many times as there are years, less by 1; then multiply this product by the annuity; and subtract the annuity therefrom; lastly, divide the remainder by the ratio less 1, and the quotient will be the amount.

EXAMPLES,

ANNUITIES or PENSIONS in ARREARS. 309

EXAMPLES.

1. What will an annuity of £60 per annum, payable yearly, amount to in 4 years, at £6 per cent. ?

First Method.— $1 + 1,06 + 1,06^2 + 1,06^3 \times 60 =$
£262 9s. 6½d.

Second Method.

$$\frac{1,06 \times 1,06 \times 1,06 \times 1,06 \times 60 - 60}{1,06 - 1} = 262,49.$$

Or, By TABLE III.

Multiply the tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount.

2. What will an annuity of £60 per annum, amount to in 20 years, allowing £6 per cent. compound interest ?

Under 6 per cent. and opposite 20, in Table 3d, you will find tabular number = 36,78559.

Multiply by 60 = Annuity.

$$2207,13540 = £ 2207 25. 8\frac{1}{2}$$

(the Ans.)

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, pension, &c.

RULE.—Multiply the whole amount by the amount of £1 for 1 year, from which subtract the whole amount ; divide the remainder by that power of the amount of £1 for 1 year, signified by the number of years, made less by unity, and the quotient will be the answer.

EXAMPLES.

310 ANNUITIES or PENSIONS in ARREARS.

EXAMPLES.

1. What annuity, being forborne 4 years, will amount to 262,47696, at £6 per cent. compound interest?

$$\frac{262,47696 \times 1,06 - 262,47696}{1,06 \times 1,06 \times 1,06 \times 1,06 - 1} = 60$$

Or, By TABLE III.

Divide the amount by the tabular number under the rate, and opposite to the time, and the quotient will be the annuity.

2. What annuity, being forborne 20 years, will amount to £2207,1354 at £6 per cent. compound interest?

$$\text{Tabular amount} = 36,78559 : 2207,1354 :: 60 \text{ Ans.}$$

CASE III.

When the annuity, amount and ratio are given, to find the time.

RULE.—Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being divided by the annuity, the quotient will be that power of the ratio signified by the time, which being divided by the amount of £1 for 1 year, and this quotient by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in Table I, and, in a line with it, you will see the time.

EXAMPLE.

1. In what time will £60 per annum, payable yearly, amount to 262,47696, allowing £6 per cent. compound interest for the forbearance of payment?

PRESENT WORTH of ANNUITIES, &c. 311

In Table I, under the given rate, you will find 1,262476, and in a line, under years, you will find 4.

PRESENT WORTH of ANNUITIES, &c. at COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given, to find the present worth.

RULE* 1.—Divide the annuity by the ratio, or the amount of £1 for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth for two years.

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the annuity sought.

Or, Divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity: This remainder being divided by the ratio less 1, the quotient will be the present worth.

E X A M P L E.

1. † What ready money will purchase an annuity

* The amount of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year,
3. Add this interest to the former account, and it will give the whole amount required.

The present worth of an annuity for years and parts of a year, may be found thus:

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

† Questions in this case may also be answered by first finding the amount of the given annuity by Case 1, of annuities in arrears, page 308, and then the present worth, or principal, by Case 2, of compound interest, page 306.

312 PRESENT WORTH of ANNUITIES

nuity of £60, to continue 4 years, at £6 per cent. compound interest?

$$\frac{60}{.06} = 47,525 \quad 60 - 47,525 = 12,475$$

$$\text{And } \frac{12,475}{.06} = 207,916$$

By TABLE III.

Under £6 per cent. and opposite 4, we find
 $4.87461 =$ Amount of £1 annuity for 4 years,
 Multiply by 60 = Annuity.

$$262,47660 = \text{Amount of } £60 \text{ for 4 years.}$$

Then opposite 4 years, and under £6 per cent. in Table II, we have, .792093 and $.792093 \times 262,4766 =$
 $(208,11974$

Or, Opposite 4 years, and under £6 per cent. in Table I, we have $1,26247 =$ the amount of £1 for 4 years : Then, $262,4766 \div 1,26247 = 208,0219 =$
 $£208 \text{ 2s. 5d. the Answer.}$

By TABLE IV.

Multiply the tabular number, under the rate, and opposite the time, into the annuity, and the product will be the present worth.

Thus, in Example 1st. What ready money will purchase £60 annuity, to continue 4 years, at £6 per cent. compound interest?

Under £6 per cent. and even with 4 years,

$$\text{We have } 3.4651 = \text{present worth } £1 \text{ for 4 years.}$$

$$\text{Mult. by } 60 = \text{annuity.}$$

$$\text{Ans.} = 207,9060 = £207 \text{ 18s. 11d.}$$

C A S E 11.

When the present worth, time and rate are given, to find the annuity, rent, &c.

RULE 1.—From that power of the ratio, denoted by the number of years plus 1, subtract that power of it, denoted by the number of years.

2. Divide the remainder by that power of the ratio, signified by the time made less by unity.

3. Multiply the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the present worth.

2. Multiply that power of the ratio, denoted by the time, by the present worth, and subtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made less by unity, and the quotient will be the annuity.

E X A M P L E.

1. What annuity, to continue 4 years, will £207,904 purchase, compound interest, at £6 per cent.?

First Method.

From $1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 = 1,3382255776$

Subt. $1,06 \times 1,06 \times 1,06 \times 1,06 = 1,26247696$

Divide by $1,06 - 1 = ,26247696$, 0757486176 , $(2885898$

And $,2885898 \times 207,9 = £60$

Second Method.

From $1,06 \times 1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 = 278,2170975731$

Take $1,06 \times 1,06 \times 1,06 \times 1,06 \times 207,9 = 262,46899984$

Divide by $1,06 - 1 = ,26247696$, $15,748137589$

$(59,998 = £60$

D d

314 ANNUITIES, &c, IN REVERSION,

By TABLE V.

Multiply the tabular number, corresponding with the rate and time, by the purchase money, and the product will be the annuity.

Under £6 per cent. and opposite 4 years, you will find, 28859 = annuity which £1 will purchase in 4 years; and, $28859 \times 207.9 = £60$.

CASE III.

When the annuity, present worth, and ratio, are given, to find the time.

RULE.—Divide the annuity by the product of the present worth and ratio subtracted from the sum of the present worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which being divided by the ratio, and this quotient by the same, until nothing remain, the number of divisions will shew the time:—Or, the above quotient being sought in Table I, under the given rate, in a line with it, you will see the time.

EXAMPLE.

For how long may an annuity of £60 per annum be purchased for £207,906336762 at £6 per cent. compound interest?

60

$$\frac{207,906336762 + 60 - 207,906336762 \times 1.06}{60} = 1.26247696$$

which being sought in Table I, under the given rate, in a line with it, is 4 = 4 years.

ANNUITIES, LEASES, &c. taken in REVERSION, at COMPOUND INTEREST.

CASE I.

When the annuity, time and ratio are given, to find the present worth of the annuity in reversion

RULE 1.—Divide the annuity by that power of the ratio, denoted by the time of its continuance.

2, Subtract

2. Subtract this quotient from the annuity : Divide the remainder by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reversion (or, time to come, before the annuity commences) and the quotient will be the present worth of the annuity in reversion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend,

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it, denoted by the time of reversion, and the ratio less 1 continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

EXAMPLES.

1. What is the present worth of £60, payable yearly, for 4 years ; but not to commence until 2 years hence, at £6 per cent. ?

In Table IVth, find the present value of £1 at the given rate, both for the time in being and the time in reversion added together, and subtract the present worth of the time in being from the other, multiply the remainder by the annuity, and the product will be the answer.

Present worth of time in being and reversion = 4,91732

Present worth of the time in being = 1,8333

3,08402

60

£185,04120

60 — 47,5256

Then, $\frac{60 - 47,5256}{1,06 - 1} = 107,906$

1,36247696

207,906

And $\frac{1,36247696}{1,1236} = 185,035876$

1,1236

Or, $\frac{1,06^4 - 1}{1,06^4 \times 1,06^2 \times 1,06 - 1} \times 60 = 185,036$

1,06^4 - 1

816 ANNUITIES, &c. IN REVERSION;

An annuity, several times in reversion, and rate, being given, to find the several present values.

Find the present value of £1 per Table IV, at the given rate, and for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

2. A has a term of 6 years in an estate of £60 per annum. B has a term of 14 years in the same estate, in reversion, after the six years are expired; and C hath a farther term of 16 years, after the expiration of 20 years: I demand the present values of the several terms, at 6 per cent. ?

£. s. d.
 Pres. val. of 1. for 36 yrs. = 14,627.22 × 60 = 877 0 72
 Ditto of ditto for 20 = 11,469.91 × 60 = 688 3 104
 Ditto of ditto for 6 = 4,917.32 × 60 = 295 0 91 = A's term.
 Therefore, 877 0 72 - 688 3 104 = £188 16 9 = C's term, and
 688 3 104 - 295 0 91 = £393 3 14 = B's term.

3. For a lease of certain profits for 7 years, A offers to pay £300 gratuity, and £300 per annum; B offers £800 gratuity and £250 per annum; C bids £1300 gratuity and £200 per annum, and D bids £2500 for the whole purchase, without any yearly rent; Which is the best offer, computing at 6 per cent. ?

By Table IV, the present worth of £300 } 1674.714
 per annum, for 7 years, at 6 per cent. is }
 To which add 300

Value of A's offer = 1974.714

Present worth of £250 per ann. for 7 years = 1395.595
 To which add 800

Value of B's offer = 2195.595

Present

Present worth of £200 per ann. for 7 years = 1116,476

To which add 1800

Value of C's offer = 2416,476

D's offer = 2500

Hence it appears that D's offer is the best.

The above questions may be answered by the 4th and 2d Tables. Take Question 1st for example.

1. Multiply the tabular number in Table IV, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table II, corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In table 4th we have 3,4651

Multiply by 60 = Annuity.

207,9060

In Table 2d we have, 889996 and $207,906 \times 889996 = 185,0351$
(Present worth of the reversion.)

CASE II.

When the present worth of the reversion, rate and time are given, to find the annuity.

RULE 1.—Multiply that power of the ratio signified by the time of reversion, by the present worth, and the product will be the amount of the present worth for the time before the annuity commences.

2. Multiply that power of the ratio signified by the time of continuance plus 1 by the last product.

3. Multiply that power of the ratio signified by the time by the aforesaid product, and this last product.

product, divided by that power of the ratio denoted by the time, minus unity, will give the annuity.

Or, Divide the continual product of the present worth, that power of the ratio denoted by the time of continuance, that power of it denoted by the time of reversion, and the ratio minus 1, by that power of the ratio denoted by the time of continuance minus 1, and the quotient will be the annuity,

EXAMPLES.

Q1. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for £185,035 876 at 6 per cent.?

Method 1.— $185,036 \times 1,1236 = 207,906$

Then $207,906 \times 1,33822 = 207,906 \times 1,26247$

$\frac{207,906 \times 1,26247}{1,26247 - 1} = £60$

Or, by Table IV. Divide the present worth of the reversion by the difference between the present worth of £1 for the time both in being and reversion, and the time in being, and the quotient will be the annuity.

Method 2. $185,036 \times 1,26247 \times 1,1236 \times 1,06 = 1$

Q2. The present worth of a lease of a house is £331 251 74, taken in reversion for 20 years, but not to commence until the end of 8 years, allowing 6 per cent. to the purchaser; What is the yearly rent?

Method 1. Multiply that power of the ratio denoted by the time of continuance first by the last pro-

duct, and then multiply that power of the ratio denoted by the time of reversion, and the last product by the ratio minus 1, and the quotient will be the annuity.

TABLE

T A B L E S.

73

TABLE FIRST.

Shewing the Amount of £1 from one Year to forty Years.

ys.	5 per cent.	6 per cent.	ys.	5 per cent.	6 per cent.
1	1,0500000	1,0600000	21	2,785625	3,3995686
2	1,1025000	1,1236000	22	2,9252607	3,6035374
3	1,1576250	1,1910160	23	3,0715237	3,8197496
4	1,2155062	1,2624769	24	3,2250909	4,0489346
5	1,2762815	1,3382250	25	3,3863549	4,2918707
6	1,3400956	1,4185191	26	3,5555076	4,5498829
7	1,4071004	1,5036302	27	3,7334563	4,8228459
8	1,4774554	1,5938480	28	3,9201201	5,1116867
9	1,5512282	1,6894780	29	4,11561356	5,4168379
10	1,6285946	1,7908426	30	4,3219413	5,7424912
11	1,7103298	1,8982985	31	4,5390394	6,0881007
12	1,7958562	2,0121964	32	4,7670414	6,4533807
13	1,8856491	2,1329282	33	5,0060885	6,8405899
14	1,9799216	2,2609039	34	5,2563472	7,2510223
15	2,0789281	2,3965581	35	5,5180152	7,6860866
16	2,1828745	2,5477193	36	5,7920101	8,1462248
17	2,2919123	2,7097727	37	6,0781409	8,6320071
18	2,4066102	2,8853391	38	6,37651772	9,1445523
19	2,52769502	3,0755995	39	6,6875111	9,6835074
20	2,6532977	3,2807355	40	7,0209087	10,2507178
100	100	100	100	100	100

LEAT

TABLE

T A B L E S.

TABLE SECOND.

Shewing the present Value of £1 due at the end of any number of Years, from 1 to 40.

years.	5pr.cent.	6pr.cent.	years.	5pr.cent.	6pr.cent.
1	,952381	,943896	21	,358942	,294135
2	,90703	,889996	22	,34185	,277505
3	,863838	,889619	23	,325571	,261797
4	,822702	,792093	24	,310068	,246978
5	,783526	,747258	25	,305303	,232998
6	,746215	,70496	26	,281241	,21981
7	,710681	,665057	27	,267848	,207368
8	,676839	,627412	28	,255094	,19503
9	,644609	,594898	29	,242946	,184556
10	,613913	,558394	30	,231377	,17411
11	,584679	,52787	31	,220359	,164255
12	,556837	,496969	32	,209806	,154957
13	,530821	,468339	33	,199874	,146186
14	,506068	,442301	34	,190355	,137912
15	,481017	,417265	35	,18119	,130105
16	,456811	,392047	36	,172057	,122741
17	,436297	,371261	37	,164436	,115793
18	,415521	,350343	38	,156605	,109182
19	,395714	,330513	39	,149148	,103055
20	,376889	,311804	40	,142046	,097222

TABLE THIRD.

Shewing the amount of £1 Annuity for any number of Years, from 1 to 40.

ys.	5 per cent.	6 per cent.	ys.	5 per cent.	6 per cent.
1	1,	1,	21	35,719262	39,992725
2	2,05	2,06	22	38,505214	42,392289
3	3,1525	3,1836	23	41,430476	46,995826
4	4,310125	4,374616	24	44,501999	50,815576
5	5,525691	5,637093	25	47,727099	54,86451
6	6,801913	6,975318	26	51,113454	59,156381
7	8,142008	8,393837	27	54,669126	63,705763
8	9,549109	9,897467	28	58,405389	68,528109
9	11,026564	11,491315	29	62,322712	73,639796
10	12,577892	13,180794	30	66,438827	79,058183
11	14,206787	14,971642	31	70,76079	84,803674
12	15,917126	16,86994	32	75,298829	90,89776
13	17,712983	18,882132	33	80,063771	97,343161
14	19,598632	21,015064	34	85,066959	104,153751
15	21,578563	23,275968	35	90,329307	111,434776
16	23,657492	25,672527	36	95,86322	119,110269
17	25,840366	28,212879	37	101,628139	127,268114
18	28,132385	30,905652	38	107,709646	135,994201
19	30,529004	33,759991	39	114,095025	145,038153
20	33,065954	36,78559	40	120,799774	154,561961

TABLE FOURTH.

Shewing the present Worth of £1 Annuity, for any number of Years, from 1 to 40.

Yrs.	5 per cent.	6 per cent.	Yrs.	5 per cent.	6 per cent.
1	0,95238	0,94339	21	12,82115	11,76407
2	1,85941	1,83339	22	13,163	12,04158
3	2,72325	2,67301	23	13,48807	12,30838
4	3,54595	3,4651	24	13,79864	12,55735
5	4,32948	4,21236	25	14,09394	12,78335
6	5,07569	4,91732	26	14,37518	13,00316
7	5,78637	5,68238	27	14,64303	13,21053
8	6,46321	6,30979	28	14,89813	13,40616
9	7,10782	6,80169	29	15,14107	13,59072
10	7,72178	7,36008	30	15,37245	13,76483
11	8,30641	7,88687	31	15,59282	13,92908
12	8,86325	8,38384	32	15,80268	14,08398
13	9,39357	8,85268	33	16,00255	14,22917
14	9,89864	9,29498	34	16,1929	14,36613
15	10,37966	9,71225	35	16,37419	14,49533
16	10,83777	10,10569	36	16,54685	14,61722
17	11,27407	10,47726	37	16,71129	14,73211
18	11,68958	10,8276	38	16,86789	14,84048
19	12,08532	11,15811	39	17,01704	14,9427
20	12,46231	11,46992	40	17,15909	15,03913

ALLIGATION.

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TABLE FIFTH.

The Annuity which £1 will purchase for any number of Years to come, from 1 to 40.

yrs.	5 per cent.	6 per cent.	yrs.	5 per cent.	6 per cent.
1	1,05	1,06	21	,078	,085
2	,5378	,54544	22	,07597	,08303
3	,36721	,37411	23	,07414	,08128
4	,28201	,28859	24	,07247	,07968
5	,28097	,28739	25	,07095	,07823
6	,19702	,20386	26	,06956	,0769
7	,17282	,17913	27	,06829	,0757
8	,15473	,16103	28	,06712	,07459
9	,14069	,14702	29	,06604	,07358
10	,1295	,13587	30	,06505	,07272
11	,12039	,12679	31	,06413	,07179
12	,11282	,11927	32	,06328	,071
13	,10645	,11296	33	,06249	,07027
14	,10102	,10758	34	,06175	,06959
15	,09624	,10296	35	,06107	,06899
16	,09227	,09895	36	,06043	,06839
17	,0887	,09544	37	,05984	,06785
18	,08555	,09285	38	,05928	,06735
19	,08274	,08962	39	,05876	,06689
20	,08024	,08718	40	,05828	,06646

ALLIGATION

Is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality: It consists of two kinds, viz, Alligation Medial, and Alligation Alter-

ALLIGATION

ALLIGATION MEDIAL

Is when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

RULE.—As the sum of the quantities, or the whole composition, is to their total value; so is any part of the composition to its mean price or value.

EXAMPLES.

1. A tobaccoist would mix 60 lb of tobacco, at 6d. per lb, with 50 lb at 1s. 40 lb at 1/6, and 30 lb at 2s. per lb—What is 1 lb of this mixture worth?

lb.	s.	d.	£.	s.	lb.	£.	lb.
60 at	0	6	11	10	As 180 :	10 :: 1	
50—	1	0	—	2		1	
40—	1	6	—	3		—	
30—	2	0	—	3		20	
						20	
Ans. of 180 lb. = £ 10 0						—	
						180) 200 (1s.	
						180	
						—	
						20	
						12	
						—	
						180) 240 (1 2/3 d.	
						s. d.	
						180 Ans. 1 1 1/3 pr. lb.	

2. A farmer would mix 20 bushels of wheat, at 6s. per bushel; 16 bushels of rye, at 4s. per bushel; 12 bushels of barley, at 3s. per bushel, and 8 bushels of oats, at 2s. per bushel: What is the value of 1 bushel of this mixture? Ans, 4s. 2 1/2 d.

3. A

ALLIGATION ALTERNATE.

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3. A wine merchant mixes 12 gallons of wine, at 4s. 10d. per gallon, with 24 gallons, at 5s. 6d. and 16 gallons, at 6s. 3d.—What is a gallon of this composition worth? *Ans.* 5s. 7d.

4. A goldsmith melted together 8 oz. of gold of 22 carats fine, 1 lb 8 oz. of 21 carats fine, and 10 oz. of 18 carats fine: Pray what is the quality, or fineness, of the composition?

$$\frac{8 \times 22 + 20 \times 21 + 10 \times 18}{8 + 20 + 10} = 20 \frac{1}{3} \text{ carats fine, } \textit{Ans.}$$

5. A refiner melts 5 lb of gold of 20 carats fine, with 8 lb of 18 carats fine: How much alloy must he put to it, to make it 22 carats fine?

$$22 - \frac{5 \times 20 + 8 \times 18}{5 + 8} = 3 \frac{2}{3}$$

Ans. It is not fine enough by $3 \frac{2}{3}$ carats, so that no alloy must be added, but more gold.

ALLIGATION ALTERNATE*

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate: So that it is the reverse of Alligation Medial, and may be proved by it.

CASE

* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such that there is precisely as much gained by one quantity as is lost by the other: and therefore the gain and loss upon the whole, are equal, and are exactly the proposed rate.

In like manner, let the number of samples be what it may, and with how many forever each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious from the rule, that questions of this sort admit of a great variety of answers; for having found 1 answer, we may find as many more as we please by only multiplying or dividing each of the quantities found, by 2, 3, 4, &c.

If any one of the samples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

E o

C A S E I.

The whole work of this case consists in linking the extremes truly together, and taking the differences between them and the mean price, which differences are the quantities sought.

RULE 1.—Place the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase; with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, and each greater rate or price with one, or any number, of the less.

3. Place the difference between the mean price (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

E X A M P L E.

1. A merchant has spices, some at 1s. 6d. per lb., some at 2s. some at 4s. and some at 5s. per lb.—How much of each sort must he mix, that he may sell the mixture at 3s. 4d. per lb.

	d.	lb.	s.	d.		d.	lb.	s.	d.
Mean	18	30	4	6		18	30	4	6
Rate 1st	24	8	-	2		24	20	-	2
Rate 2nd	48	16	-	4		48	24	-	4
	60	24	-	6		60	16	-	6

$$40d. \left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\} \begin{array}{l} 20 + 8 \\ 8 \\ 16 + 22 \\ 22 \end{array} \left| \begin{array}{l} 28 \text{ at } 16 \\ 8 - 2 \\ 33 - 4 \\ 22 - 5 \end{array} \right\} \text{ per lb.}$$

$$40d. \left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\} \begin{array}{l} 20 \\ 8 + 20 \\ 16 \\ 22 + 16 \end{array} \left| \begin{array}{l} 20 \text{ at } 16 \\ 28 - 2 \\ 16 - 4 \\ 38 - 5 \end{array} \right\} \text{ per lb.}$$

$$40d. \left\{ \begin{array}{l} 18 \\ 24 \\ 48 \\ 60 \end{array} \right\} \begin{array}{l} 20 + 8 \\ 8 + 20 \\ 16 + 22 \\ 22 + 16 \end{array} \left| \begin{array}{l} 28 \text{ at } 16 \\ 28 - 2 \\ 38 - 4 \\ 38 - 5 \end{array} \right\} \text{ per lb.}$$

Note. These five answers arise from as many various ways of linking the rates of the ingredients together.

2. * A merchant has Canary wine, at 3s. per gallon; Sherry at 2s. 1d. and claret at 1s. 5d. per gallon: How much of each sort must he take, to sell it at 2s. 4d. per gallon?

$$\text{Mean rate } 28d. \left\{ \begin{array}{l} 36 \\ 25 \\ 17 \end{array} \right\} \begin{array}{l} 3 + 11 \\ 8 \\ 8 \end{array} \left| \begin{array}{l} 14 \text{ at } 30 \\ 8 - 21 \\ 8 - 15 \end{array} \right\} \text{ per gall.}$$

3. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine: What quantity of each must he take?

$$21 \left\{ \begin{array}{l} 16 \\ 18 \\ 19 \\ 23 \\ 24 \end{array} \right\}$$

4. It

* *Note.* The 2d question admits but one way of linking, and so but of one answer; yet all numbers in the same proportion between themselves, as the numbers which compose the answer, will likewise satisfy the condition of the question.

4. It is required to mix several sorts of wine at 3s. 5s. and 7s. per gallon, with water, that the mixture may be worth 4s. per gallon : How much of each sort must the mixture consist of ?

$$4 \left\{ \begin{array}{l} 0 \\ 3 \\ 5 \\ 7 \end{array} \right\}$$

{ *Ans.* 3 gal. water, 1 gal. wine, at 3s.
1 ditto, at 5s. and 4 ditto, at 7s.

C A S E II.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixtures are given, to find the several quantities of the rest, in proportion to the quantity given.

RULE.—Take the differences between each price, and the mean rate, and place them alternately, as in Case 1. Then, As the difference standing against that simple, whose quantity is given, is to that quantity ; so is each of the other differences, severally, to the several quantities required.

E X A M P L E S.

1. A merchant has 40lb of tea, at 6s. per lb, which he will mix with some at 5s. 8d. some at 5s. 2d. and some at 4s. 6d.—How much of each sort must he take, to mix with the 40lb, that he may sell the mixture at 5s. 5d. per lb.?

$$d. \left\{ \begin{array}{l} 54 \\ 62 \\ 68 \\ 72 \end{array} \right\} \begin{array}{l} 7+3 \\ 3+7 \\ 3+11 \\ 11+3 \end{array} \left| \begin{array}{l} 10 \\ 10 \\ 14 \\ 14 \end{array} \right. \quad \begin{array}{l} \text{[city.} \\ \text{14 stands against the given quan-} \\ \text{ty.} \end{array}$$

$$\text{As } 14 : 40 :: \left\{ \begin{array}{l} 10 : 28 \frac{1}{2} \text{ at } 4s. 6d. \\ 10 : 28 \frac{1}{2} \text{ — } 5s. 2d. \\ 14 : 40 \text{ — } 5s. 8d. \end{array} \right\} \text{ per lb.}$$

How

ALLIGATION ALTERNATE. 329

Q. How much gold, of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 10 oz. of 18 carats fine, that the composition may be 22 carats fine?

Ans. 10 oz. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

ALTERNATION TOTAL.

CASE III.

When the rates of the several ingredients, the quantity to be compounded, and the mean rate of the whole mixture are given, to find how much of each sort will make up the quantity.

RULE.—Place the differences between the mean rate, and the several prices alternately, as in Case 1st. Then, As the sum of the quantities, or differences thus determined, is to the given quantity or whole composition; so is the difference of each rate, to the required quantity of each rate.

EXAMPLES.

1. Suppose I have 4 sorts of currants, at 8d. 12d. 18d. and 22d. per lb.—the worst would not sell, and the best were too dear; I therefore concluded to mix 120lb. and so much of each sort, as to sell them at 16d. per lb.—How much of each sort must I take?

d.	lb.		lb. lb.	
	8	6		6 : 36 at 8d.
	12	2	lb. lb.	2 : 12 — 12d.
16d.	18	4	As 20 : 120 ::	4 : 24 — 18d.
	22	8		8 : 48 — 22d.
		Sum = 20		120

2. A goldsmith has several sorts of gold, viz. of 25, 17, 20, and 22 carats fine, and would melt together, of all these sorts, so much as may make a
c c
mass

SINGLE POSITION. 831

$$5^2 \left\{ \begin{array}{l} 42 \\ 54 \\ 60 \end{array} \right\} \begin{array}{l} 2+8 \\ 10 \\ 10 \end{array} \left| \begin{array}{l} 10 \\ 10 \\ 10 \end{array} \right.$$

As 10 : $\left\{ \begin{array}{l} 10 \\ 10 \end{array} \right\} :: 12 : \left\{ \begin{array}{l} 12 \text{ gallons, at } 4/6 \\ 12 \text{ gallons, at } 5s. \end{array} \right\} \text{ per gal. } \text{Ans.}$

POSITION.

Position is a rule, which, by false, or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts ; Single and Double.

SINGLE POSITION.

Single Position teaches to resolve those questions, whose results are proportional to their suppositions : Such are those which require the multiplication or division of the number sought by any proposed number ; or when it is to be increased or diminished by itself a certain proposed number of times.

RULE 1.—Take any number, and perform the same operations with it, as are described to be performed in the question.

2. Then say ; As the sum of the errors is to the given sum ; so is the supposed number to the true one required.

PROOF.—Add the several parts of the sum together, and if it agrees with the sum, it is right.

EXAMPLES.

1. A schoolmaster being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435 ; Of what number did his school consist ?

Suppose

232 SINGLE POSITION.

Suppose he had 80 As 290:435 :: 80

As many = 80	80	
$\frac{1}{2}$ as many = 60		120
$\frac{1}{3}$ as many = 40	290	120
$\frac{1}{4}$ as many = 20	29	90
$\frac{1}{5}$ as many = 10		60
	58	80
	58	15
290		
	0	485 Proof.

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6l. per cent. per annum, simple interest, and at the end of 12 years, received for principal and interest 860l.; What was the sum lent? *Ans.* 500l.

3. A, B and C joined their stocks, and gained 350l.; of which, A took up a certain sum; B took up four times so much as A, and C, eight times so much as B; What share of the gain had each?

Ans. { $\begin{array}{lll} \text{9} & \text{9} & \text{2} & \text{1} \frac{3}{4} & \text{A's share.} \\ \text{87} & \text{16} & \text{9} & \text{0} \frac{12}{17} & \text{B's ditto.} \\ \text{302} & \text{14} & \text{0} & \text{2} \frac{2}{17} & \text{C's ditto.} \end{array}$

4. A, B, C and D spent 35l. at a reckoning, and, being a little dipped, they agreed that A should pay $\frac{3}{8}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{10}$; What did each pay in the above proportion?

Ans. { $\begin{array}{ll} \text{A,} & \text{13} & \text{4} \\ \text{B,} & \text{10} & \\ \text{C,} & \text{6} & \text{8} \\ \text{D,} & \text{6} & \end{array}$

5. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{6}$, D $\frac{1}{10}$, and E the remainder, which is £40; What is the sum?

Suppose £200 then $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} = 120$.

200 - 120 = 80, As 80:40 :: 200:100, *Ans.*

6. A

6. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had £26 $\frac{2}{3}$ left; What had he at first?

Ans. £160.

7. A and B talking of their ages, B said his age was once and a half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93; What was the age of each?

Ans. A's 12, B's 18, and C's 63 years.

8. A vessel has 3 cocks, A, B and C; A can fill it in half an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour; In what time will they all fill it together?

Ans. $\frac{1}{3}$ hour.

9. A person having about him a certain number of dollars, said that $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of them would make 57; Pray, how many had he?

Ans. 60.

10. A gentleman bought a chaise, horse and harness for £100; the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{3}$ more than the horse; What was the price of each?

Ans. Har. £25 $\frac{2}{3}$, horse £31 $\frac{4}{3}$, chaise £43 $\frac{2}{3}$.

11. A and B, having found a purse of money, disputed who should have it: A said that $\frac{1}{2}$, $\frac{1}{10}$ and $\frac{1}{10}$ of it amounted to £35, and if B could tell him how much was in it he should have the whole, otherwise he should have nothing: How much did the purse contain?

Ans. £100.

12. A gentleman divided his fortune among his sons; to A he gave £9 as often as to B £5; and to C but £3 as often as to B £7, yet C's portion came to 1050 $\frac{1}{2}$; What was the whole estate?

Ans. £7916 $\frac{2}{3}$.

13. Seven eighths of a certain number exceeds four fifths by 6; What is that number?

Ans. 80.

14. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of itself, the sum will be 234 $\frac{3}{4}$?

Ans. 90.

DOUBLE

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule; such are those, in which the number sought is increased or diminished by some given number which is no known part of the number required.

RULE * 1.—Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errors against their positions or supposed

Pos. Err.

30 12

numbers, thus, $\times$ and if the error be too great

20 6

mark it with +; and if too small, with —.

3. Multiply them crosswise; that is, the first position by the last error, and the last position by the first error.

4. If the errors be alike; that is, both too small, or both too great, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors be unlike; that is, one too small, and the other too great, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note, When the errors are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

* The rule is founded on this supposition, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number; when that is not the case, the exact answer to the question cannot be found by this rule.

DOUBLE POSITION.

385

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and lining for it, at 3s. per yard; the gown and lining contained 15 yards, and the price of the whole was £3 10s.—How many yards were there of each?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards of lining, value 27s.

Sum of their values = 75s.

So that the first error is 5 too much, or + 5

Again, suppose she had 4 yards of damask, value 32s.

Then she must have 11 yards of lining, value 33s.

Sum of their values = 65s.

So that the second error is 5 too little, or — 5.

6 5+

Then, X

5 yds. at 8s. = £2 0 0

4 5—

10 yds. at 3s. = 1 10 0

20 30

£3 10 0

20

(Proof.

Sum of errors. —

= 5 + 5 = 10)50

Ans. 5 yards damask, and 10 — 5 = 5 yds.

(lining.

Or, $6 + 4 \div 2 = 5$ as before.

2. A and B have the same income: A saves $\frac{1}{3}$ of his; but B, by spending £30 per annum more than A, at the end of 8 years finds himself £40 in debt; What is their income, and, what does each spend per annum?

Ans. their income is £100

Suppose X (per annum)

160

Also, A spends £175 and

(B £105 per annum.

Then, $80 - 10 = 70$, A's expense per annum; and

70

$70 + 30 = 100$, B's expense per annum.—Then
 $100 \times 8 - 80 \times 8 = 160$, which should have been 40;
 therefore $160 - 40 = 120$ more than it should be, for
 the first error. In like manner proceed for the second error.

3. A and B laid out equal sums of money in trade: A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost £25; then A's money was double that of B; What did each lay out?

Suppose $\begin{cases} 300 & 225+ \\ & X \\ 900 & 225- \end{cases}$ *Ans.* £600.

4. A laborer was hired for 60 days upon this condition, that, for every day he wrought, he should receive 3s. 4d. and for every day he was idle should forfeit 1s. 8d. at the expiration of the time he received £3 15s.; How many days did he work, and how many was he idle?

Suppose he worked $\begin{cases} 20 & 900- \\ & X \\ 40 & 300+ \end{cases}$

Ans. He was employed 35 days, and was idle 25.

5. A gentleman has two horses of considerable value, and a carriage worth £100. Now, if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in, they will be 7 times the value of the first: What is the value of each horse?

Suppose $\begin{cases} 30 & 20- \\ & X \\ 40 & 160- \end{cases}$

Ans. One £30, and the other £40.

6. There is a fish, whose head is 10 feet long; his tail is as long as his head and half the length of his body, and his body as long as the head and tail: What is the whole length of the fish?

Ans. 100 feet.

DOUBLE POSITION. 837

First, suppose the body 20 10— Head = 10
 10— Tail = 30
 X Body = 40
 ad, suppose it 30 5—

Ans. 80 feet.

7. What number is that, which, being increased by its $\frac{1}{2}$, its $\frac{1}{3}$, and 5 more, will be doubled?

Suppose $\begin{cases} 8 & 3+ \\ X & \\ 16 & 1+ \end{cases}$

Ans. 20

8. A farmer having driven his cattle to market, received for them all, £80, being paid at the rate of £6 per ox, £4 per cow, and £1 10s. per calf; there were as many oxen as cows, and 4 times as many calves as cows: How many were there of each sort?

Suppose cows 6 16+

X

Suppose 12 112+

Ans. 5 oxen, 5 cows, and 20 calves.

9. A, B and C built a ship, which cost them £1000—of which A paid a certain sum—B paid £100 more than A, and C £100 more than both; having finished her, they fixed her for sea with a cargo worth twice the value of the ship: The out-fits and charges of the voyage amounted to $\frac{1}{4}$ of the ship; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{1}{4}$ of the vessel, cargo and expenses: Please to inform me what the ship cost them, severally; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained?

Suppose it cost A £100 300—

X

Suppose - - 200 100+

A owned $\frac{1}{4}$ of the ship, which cost him £175, and his share of the gain was £218 15s.—B owned $\frac{1}{4}$, which cost £275, and his gain was £348 15s.—C owned $\frac{1}{2}$, which cost £550, and his gain was £687 10s.

Ff

PERMUTATIONS.

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PERMUTATIONS *and* COMBINATIONS.

The Permutation of Quantities is the shewing how many different ways any given number of things may be changed.

This is also called *variation, alternation, or changes*; and the only thing to be regarded here is the order they stand in; for no two parcels are to have all their quantities placed in the same situation.

The Combination of Quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is sometimes called *election, or choice*; and here every parcel must be different from all the rest, and no two are to have precisely the same quantities or things.

The Composition of Quantities is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

Here, no regard is had to their places; and it differs from Combination, only, as that admits but of one row of things.

PROBLEM 1.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

RULE.—Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

EXAMPLES.

* Any two things *a* and *b* are capable of two variations only; as *ab, ba*; whose number is expressed by 1×2 .

If there be three things, *a, b* and *c*, then any two of them, leaving out the third, will have 1×2 variations; and consequently when the third is taken in, there will be $1 \times 2 \times 3$ variations; and so on, as far as you please,

PERMUTATIONS AND COMBINATIONS. 339

EXAMPLES.

1. Christ Church, in Boston, has 8 bells; How many changes may be rung on them?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320 \text{ the Ans.}$$

2. Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that, in a frolic, they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner; Pray how long, had they kept their agreement, would their frolic have lasted?

$$\text{Ans. } 9941 \frac{335}{365} \text{ years.}$$

3. How many changes or variations will the alphabet admit of?

$$\text{Ans. } 620448401733239439360000.$$

PROBLEM 2.

Any number of different things being given—to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE.—Take a series of numbers, beginning at the number of things given, and decreasing by 1, to the number of quantities to be taken at a time; the product of all the terms will be the answer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$8 \times 7 \times 6 \times 5 (=4 \text{ terms}) = 1680 \text{ the Ans.}$$

2. How many words can be made with 6 letters of the alphabet, admitting a number of consonants may make a word?

$$\text{Ans. } 96909120.$$

PROBLEM 3.

Any number of things being given—whereof there are several things of one sort, several of another, &c. To find how many changes may be made out of them all.

RULE 1.—Take the series $1 \times 2 \times 3 \times 4$, &c. up to the

* Any 2 quantities, a, b , both different, admit of two changes; but if the quantities are the same, or ab become aa , there will be only one alteration, which may be expressed by $\frac{1 \times 2}{1 \times 1} = 1$.

$$1 \times 1$$

Any

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the number of things given, and find the product of all the terms,

2. Take the series $1 \times 2 \times 3 \times 4$, &c. up to the number of the given things of the first sort, and the series, $1 \times 2 \times 3 \times 4$, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the joint product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15$ (= number of letters in the word) = 1307674368000.

$1 \times 2 \times 3 \times 4 \times 5$ (= number of z's) = 120

1×2 (= number of p's) = 2

1 (= number of t's) = 1

$1 \times 2 \times 3$ (= number of h's) = 6

1×2 (= number of n's) = 2

$1 \times 6 \times 1 \times 2 \times 120 = 1440$ (1307674368000/1440 = 907674368000—the Ans.)

2. How many different numbers can be made of the following figures, 1223334444? *Ans.* 12600.

PROBLEM 4.

To find the number of combinations of any given number of things, all different from one another, taking one given number at a time.

RULE 1.—Take the series 1, 2, 3, 4, &c. up to the

Any 3 quantities, a, b, c , all different from each other, admit of 6 variations; but if the quantities are all alike, or a, b, c become aaa , then the 6 variations will be reduced to 1, which may be expressed

by $\frac{1 \times 2 \times 3}{1 \times 2 \times 3} = 1$. Again, if two quantities out of three are alike,

or, abc , become aac ; then the 6 variations will be reduced to these 3, aac, caa, aca , which may be expressed by $\frac{1 \times 2 \times 3}{1 \times 2} = 3$ and so of any greater number.

* In any given number of quantities, the number of combinations increases gradually until you come about the mean numbers, and

PERMUTATIONS AND COMBINATIONS. 341

the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?

$$\begin{aligned} 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 & (= \text{the number to be taken at a time,}) \\ 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 & (= \text{same number from 12}) \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} = 5040$$

$$5040 \div 3991680 = 792 \text{ the Ans.}$$

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet?

$$\text{Ans. } 134596.$$

3. A general was asked by his king, what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men: What did it amount to?

$$\text{Ans. } £2383602841 \frac{7}{11}$$

4. A farmer bargained with a gentleman for a dozen sheep (at 2 dollars per head) which were to be picked out of a dozen; but, being long in choosing them, the gentleman told him, that if he would give him a penny for every different dozen which might be chosen out of the two dozen, he should have the whole, to which the farmer readily agreed: Pray what did they cost him?

$$\text{Ans. } £11267 \frac{6}{4}$$

PROBLEM

and then gradually decreases. If the number of quantities be even, half the number of places will shew the greatest number of combinations, that can be made of those quantities; but if odd, then those two numbers, which are the middle, and whose sum is equal to the given number of quantities, will shew the greatest number of combinations.

ff.

342 The USE of LOGARITHMS.

PROBLEM 5.

To find the compositions of any number, in an equal number of sets, the things being all different.

RULE.—Multiply the number of things in every set continually together, and the product will be the answer required.

EXAMPLES.

1. Suppose there are 5 companies, each consisting of 12 men: It is required to find how many ways 9 men may be chosen, one out of each company? $9 \times 9 \times 9 \times 9 \times 9 = 59049$ different ways.

2. How many chances are there in throwing 4 dice? As a die has 6 sides, multiply 6 into itself three times continually. $6 \times 6 \times 6 \times 6 = 1296$ chances, *Ans.*

3. Suppose a man undertakes to throw an ace at one throw with 4 dice; what is the probability of his effecting it?

First, $6 \times 6 \times 6 \times 6 = 1296$ different ways with and without the ace;

Then, if we exclude the ace side of the die, there will be five sides left, and $5 \times 5 \times 5 \times 5 = 625$, ways without the ace; therefore there are $1296 - 625 = 671$ ways, wherein one or more of them may turn up an ace: And the probability that he will do it, as 671 to 625, *Ans.*

The USE of LOGARITHMS.

1. IN MULTIPLICATION.

Given two numbers, viz. 275 and 12,6 to find their product.

RULE.—To the logarithm of 275, viz. 2,43938

Add the logarithm of 12,6, viz. 1,10087

And their sum is the logarithm of their product, viz. $3465 = 3,53925$

2. IN DIVISION.

Let it be required to find the quotient, which arises by dividing one number by another; suppose 1425 by 57.

The USE of LOGARITHMS. 343

From the logarithm of the divid. viz. $1425 = 3,15381$

Take the logarithm of the divisor, viz. $57 = 1,75587$

And the remainder is the loga- } _____

rithm of the quotient, viz. } $25 = 1,39794$

3. In the RULE of THREE.

Three numbers given to find a fourth, in direct proportion.

RULE.—From the Tables take the logarithms of each of the proposed numbers, then, add the logarithms of the second and third together, and from the sum take the logarithm of the first, and the remainder will be the logarithm of the fourth number.

Let the three proposed numbers be 18, 24 and 33, and the operation will stand thus ;

$1,38021 =$ the logarithm of 24, the 2d term.

$1,51851 =$ the logarithm of 33, the 3d term.

$2,89872 =$ the logarithm of their product.

$-1,25527 =$ the logarithm of the first term 18.

$1,64345 =$ the logarithm of the 4th term required, which, by the Table, answers to the natural number 44, the 4th proportional to the three proposed numbers.

4. In INVOLUTION or RAISING POWERS.

To find any power of any proposed number, or to involve any number to any proposed power, by logarithms.

RULE.—Multiply the logarithm of the given root by the power, viz. by 2 for the square, by 3 for the cube, &c. and the product is the logarithm of the power sought.

Required to find the cube of 12 ?

$1,07918 =$ the logarithm of 12.

$\times 3 =$ the third power, or cube.

$3,23754 = 1728$, the cube of 12.

5. In EVOLUTION, or EXTRACTING ROOTS.

To Extract any Root of any proposed number.

RULE.—Divide the logarithm of the proposed number by the index of the required root, viz. by 2 for the square, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required:

344 The USE of LOGARITHMS.

Required to find the cube root of 1728?
 $3,23754 =$ the logarithm of 1728, and $3,23754 \div 3 = 1,07918$ is the logarithm of the cube root of 1728, viz. 12.

6. In COMPOUND INTEREST.

To find the amount of any sum for any time, and at any rate, at compound interest.

RULE.—Multiply the logarithm of the ratio (i. e. the amount of 1*l*. for 1 year) by the number of years, and to the product add the logarithm of the principal; the sum will be the logarithm of the amount.

What will 45*l*. amount to, forborn 12 years, at 6 per cent. per annum, compound interest?

Log. of 1,06, the ratio, is ,02533

Multiply by the time 12

—————
 ,30396

Add log. of 45, the principal 1,65321

—————
 The sum is 1,95717 which is the logarithm of 90,7 = £ 90 14*s*. Ans.

7. In DISCOUNT, at COMPOUND INTEREST.

To find the present worth of any sum of money, due any time hence, at any rate, at compound interest.

RULE.—From the logarithm of the sum to be discounted, subtract the logarithm of the rate multiplied by the time; and the remainder is the logarithm of the present worth.

What present money will pay a debt of 90*l*. 14*s*. due 12 years hence, discounting at the rate of 6 per cent. per annum?

From the logarithm of £ 90 14 = 1,95717

Sub. product of the Log. of the ratio $\times$ by the time } = ,30396

—————
 The remainder 1,65421 is the logarithm of £ 45 Ans.

N. Hamp.

TABLE OF EXCHANGE. 343

N. Hamp- Massachu- R. Island, Connecti- cut and Virginia.				Federal Coin.			New York, and Northcare- lina.			New Jersey, Pennsylva- nia, Dela- ware and Maryland.			South Carolina and Georgia.			French Money.	
£.	s.	d.	DoL &c.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	Livres. Tour.	Sous.
	1		0,01 $\frac{7}{8}$			1 $\frac{1}{3}$			1 $\frac{1}{4}$			1 $\frac{1}{2}$			7 $\frac{1}{2}$		1 $\frac{1}{4}$
	2		0,02 $\frac{1}{2}$			2 $\frac{2}{3}$			2 $\frac{1}{2}$			2 $\frac{1}{2}$			1 $\frac{1}{2}$		2 $\frac{1}{4}$
	3		0,04 $\frac{1}{2}$			4			3 $\frac{1}{2}$			3 $\frac{1}{2}$			2 $\frac{1}{2}$		4 $\frac{1}{2}$
	4		0,05 $\frac{1}{2}$			5 $\frac{1}{3}$			5			5			3 $\frac{1}{2}$		5 $\frac{1}{2}$
	5		0,06 $\frac{1}{2}$			6 $\frac{2}{3}$			6 $\frac{1}{2}$			6 $\frac{1}{2}$			3 $\frac{1}{2}$		7 $\frac{1}{4}$
	6		0,08			8			7 $\frac{1}{2}$			7 $\frac{1}{2}$			4 $\frac{1}{2}$		8 $\frac{1}{2}$
	7		0,09 $\frac{1}{2}$			9 $\frac{1}{3}$			8 $\frac{1}{2}$			8 $\frac{1}{2}$			5 $\frac{1}{2}$		10 $\frac{1}{4}$
	8		0,11 $\frac{1}{2}$			10 $\frac{2}{3}$			10			10			6 $\frac{1}{2}$		11 $\frac{1}{2}$
	9		0,12 $\frac{1}{2}$			10			11 $\frac{1}{2}$			11 $\frac{1}{2}$			7		13 $\frac{1}{2}$
	10		0,13 $\frac{1}{2}$			11 $\frac{1}{3}$			10 $\frac{1}{2}$			10 $\frac{1}{2}$			7 $\frac{1}{2}$		14 $\frac{1}{2}$
	11		0,15 $\frac{1}{2}$			12 $\frac{1}{3}$			11 $\frac{1}{4}$			11 $\frac{1}{4}$			8 $\frac{1}{2}$		16 $\frac{1}{4}$
1	0		0,16 $\frac{1}{2}$			12			12			12			9 $\frac{1}{2}$		17 $\frac{1}{2}$
2	0		0,33 $\frac{1}{2}$			24			24			24	1	0	0 $\frac{1}{2}$	1	15
3	0		0,50			40			36			36	2	4	0	2	12 $\frac{1}{2}$
4	0		0,66 $\frac{2}{3}$			54			50			50	3	12	0	3	10
5	0		0,83 $\frac{1}{3}$			72			68			68	4	10	0	4	7 $\frac{1}{2}$
6	0		1,00			96			90			90	5	8	0	5	5
7	0		1,16 $\frac{2}{3}$			128			120			120	6	4	0	6	2 $\frac{1}{2}$
8	0		1,33 $\frac{1}{3}$			160			160			160	7	0	0	7	0
9	0		1,50			192			184			184	8	0	0	8	17 $\frac{1}{2}$
10	0		1,66 $\frac{2}{3}$			224			216			216	9	0	0	9	15
1	0	0	3,33 $\frac{1}{3}$	1	6	8			30			30	15	6	0	17	10
2	0	0	6,66 $\frac{2}{3}$	2	13	4			60			60	30	12	0	35	
3	0	0	10,00	4	0	0			90			90	45	18	0	52	10
4	0	0	13,33 $\frac{1}{3}$	5	6	8			120			120	60	24	0	70	
5	0	0	16,66 $\frac{2}{3}$	7	2	0			150			150	90	36	0	105	
6	0	0	20,00	9	6	0			180			180	120	48	0	140	
7	0	0	23,33 $\frac{1}{3}$	11	13	0			210			210	150	60	0	175	
8	0	0	26,66 $\frac{2}{3}$	14	0	0			240			240	180	72	0	210	
9	0	0	30,00	17	6	0			270			270	210	84	0	245	
10	0	0	33,33 $\frac{1}{3}$	20	13	0			300			300	240	96	0	280	

Federal

TABLE of EXCHANGE.

Federal Money.	N. Hampshire, R. Island, Connecticut & Virginia.	New York and N. Carolina.	New Jersey, Pennsylvan. Delaware, and Maryland.	S. Carolina and Georgia.	French.
D. d. c.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	Liver. Tour.
0,01	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$	$\frac{1}{10}$
0,02	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
0,03	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{3}{10}$
0,04	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
0,05	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
0,06	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{3}{5}$
0,07	$\frac{7}{10}$	$\frac{7}{10}$	$\frac{7}{10}$	$\frac{7}{10}$	$\frac{7}{10}$
0,08	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
0,09	$\frac{9}{10}$	$\frac{9}{10}$	$\frac{9}{10}$	$\frac{9}{10}$	$\frac{9}{10}$
0,10	1	1	1	1	1
0,20	2	2	2	2	2
0,30	3	3	3	3	3
0,40	4	4	4	4	4
0,50	5	5	5	5	5
0,60	6	6	6	6	6
0,70	7	7	7	7	7
0,80	8	8	8	8	8
0,90	9	9	9	9	9
1,00	10	10	10	10	10
2,00	20	20	20	20	20
3,00	30	30	30	30	30
4,00	40	40	40	40	40
5,00	50	50	50	50	50
6,00	60	60	60	60	60
7,00	70	70	70	70	70
8,00	80	80	80	80	80
9,00	90	90	90	90	90
10,00	100	100	100	100	100
20,00	200	200	200	200	200
30,00	300	300	300	300	300
40,00	400	400	400	400	400
50,00	500	500	500	500	500

A T

d.

 $\frac{1}{10}$ $\frac{1}{5}$ $\frac{1}{2}$ $\frac{3}{4}$

1

 $1\frac{1}{4}$ $1\frac{1}{2}$ $1\frac{3}{4}$

2

 $2\frac{1}{4}$ $2\frac{1}{2}$ $2\frac{3}{4}$

3

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8

 $8\frac{1}{4}$ $8\frac{1}{2}$ $8\frac{3}{4}$

9

 $9\frac{1}{4}$ $9\frac{1}{2}$ $9\frac{3}{4}$

T A B L E S.

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A TABLE, directing how to buy and sell by the Hundred.

d.	l. s. d.	s. d.	l. s. d.	s. d.	l. s. d.
$\frac{1}{4}$	0 2 4	8 $\frac{1}{4}$	3 17 0	1 4 $\frac{1}{4}$	7 11 8
$\frac{1}{2}$	0 4 8	8 $\frac{1}{2}$	3 19 4	1 4 $\frac{1}{2}$	7 14 0
$\frac{3}{4}$	0 7 0	8 $\frac{3}{4}$	8 1 8	1 4 $\frac{3}{4}$	7 16 4
1	0 9 4	9	4 4 0	1 5	7 18 8
$1\frac{1}{4}$	0 11 8	9 $\frac{1}{4}$	4 6 4	1 5 $\frac{1}{4}$	8 1 0
$1\frac{1}{2}$	0 14 0	9 $\frac{1}{2}$	4 8 8	1 5 $\frac{1}{2}$	8 3 4
$1\frac{3}{4}$	0 16 4	9 $\frac{3}{4}$	4 11 0	1 5 $\frac{3}{4}$	8 5 8
2	0 18 8	10	4 13 4	1 6	8 8 0
$2\frac{1}{4}$	1 1 0	10 $\frac{1}{4}$	4 15 8	1 6 $\frac{1}{4}$	8 10 4
$2\frac{1}{2}$	1 3 4	10 $\frac{1}{2}$	4 18 0	1 6 $\frac{1}{2}$	8 12 8
$2\frac{3}{4}$	1 5 8	10 $\frac{3}{4}$	5 0 4	1 6 $\frac{3}{4}$	8 15 0
3	1 8 0	11	5 2 8	1 7	8 17 4
$3\frac{1}{4}$	1 10 4	11 $\frac{1}{4}$	5 5 0	1 7 $\frac{1}{4}$	8 19 8
$3\frac{1}{2}$	1 12 8	11 $\frac{1}{2}$	5 7 4	1 7 $\frac{1}{2}$	9 2 0
$3\frac{3}{4}$	1 15 0	11 $\frac{3}{4}$	5 9 8	1 7 $\frac{3}{4}$	9 4 4
4	1 17 4	12	5 12 0	1 8	9 6 8
$4\frac{1}{4}$	1 19 8	1 0 $\frac{1}{4}$	5 14 4	1 8 $\frac{1}{4}$	9 9 0
$4\frac{1}{2}$	2 2 0	1 0 $\frac{1}{2}$	5 16 8	1 8 $\frac{1}{2}$	9 11 4
$4\frac{3}{4}$	2 4 4	1 0 $\frac{3}{4}$	5 19 0	1 8 $\frac{3}{4}$	9 13 8
5	2 6 8	1 1	6 1 4	1 9	9 16 0
$5\frac{1}{4}$	2 9 0	1 1 $\frac{1}{4}$	6 3 8	1 9 $\frac{1}{4}$	9 18 4
$5\frac{1}{2}$	2 11 4	1 1 $\frac{1}{2}$	6 6 0	1 9 $\frac{1}{2}$	10 0 8
$5\frac{3}{4}$	2 13 8	1 1 $\frac{3}{4}$	6 8 4	1 9 $\frac{3}{4}$	10 3 0
6	2 16 0	1 2	6 10 8	1 10	10 5 4
$6\frac{1}{4}$	2 18 4	1 2 $\frac{1}{4}$	6 13 0	1 10 $\frac{1}{4}$	10 7 8
$6\frac{1}{2}$	3 0 8	1 2 $\frac{1}{2}$	6 15 4	1 10 $\frac{1}{2}$	10 10 0
$6\frac{3}{4}$	3 3 0	1 2 $\frac{3}{4}$	6 17 8	1 10 $\frac{3}{4}$	10 12 4
7	3 5 4	1 3	7 0 0	1 11	10 14 8
$7\frac{1}{4}$	3 7 8	1 3 $\frac{1}{4}$	7 2 4	1 11 $\frac{1}{4}$	10 17 0
$7\frac{1}{2}$	3 10 0	1 3 $\frac{1}{2}$	7 4 8	1 11 $\frac{1}{2}$	10 19 4
$7\frac{3}{4}$	3 12 4	1 3 $\frac{3}{4}$	7 7 0	1 11 $\frac{3}{4}$	11 1 8
8	3 14 8	1 4	7 9 4	2 0	11 4 0

T A B L E S.

A TABLE for reducing Troy Weight to Avoirdupois.

Troy.		Avoirdupois.		Troy.	
lb.		lb.	oz. drams.	part.	drams.
4000	3291	6	13,68	19	16,67
3000	2528	9	2,26	18	15,79
2000	1645	11	6,84	17	14,92
1000	822	13	1,42	16	14,04
900	740	9	2,28	15	13,16
800	658	4	9,14	14	12,29
700	576	0	0,	13	11,41
600	493	11	6,85	12	10,53
500	411	6	13,71	11	9,65
400	329	2	4,57	10	8,78
300	246	13	11,42	9	7,9
200	164	9	2,28	8	7,02
100	82	4	9,15	7	6,14
90	74	0	13,62	6	5,27
80	65	13	4,11	5	4,39
70	57	9	9,6	4	3,51
60	49	5	15,08	3	2,63
50	41	2	4,57	2	1,75
40	32	14	10,05	1	0,88
30	24	10	15,54	gr.	
20	16	7	5,03	23	,84
10	8	3	10,52	22	,8
9	7	6	7,86	21	,77
8	6	9	5,21	20	,73
7	5	12	2,56	19	,69
6	4	14	15,9	18	,66
5	4	1	13,25	17	,62
4	3	4	10,6	16	,58
3	2	7	7,95	15	,55
2	1	10	5,3	14	,51
1	0	13	2,65	13	,47
oz.	10	8	1,09	12	,44
11	11	12	1,09	11	,4
10	10	10	15,54	10	,36
9	9	9	13,99	9	,33
8	8	8	12,43	8	,29
7	7	7	10,88	7	,26
6	6	6	9,32	6	,22
5	5	5	7,77	5	,18
4	4	4	6,22	4	,15
3	3	3	4,66	3	,11
2	2	2	3,11	2	,07
1	1	1	1,55	1	,04

A TABLE for reducing Avoirdupois Weight to Troy Weight.

Avoir.	Troy.				Avoir.	Troy.			
lb.	lb.	oz.	pwt.	gr.	oz.	lb.	oz.	pwt.	gr.
6000	7391	8	0	0	15	11	1	13	10,5
5000	6076	4	13	8	14	11	0	15	5
4000	4861	1	6	16	13	11	16	23,5	
3000	3645	10	0	0	12	10	18	18	
2000	2430	6	13	8	11	10	0	12,5	
1000	1215	3	6	16	10	9	2	7	
900	1093	9	0	0	9	8	4	1,5	
800	972	2	13	8	8	7	5	20	
700	850	8	6	16	7	6	7	14,5	
600	729	2	10	0	6	5	9	9	
500	607	7	13	8	5	4	11	8,5	
400	486	1	6	16	4	3	12	22	
300	364	7	0	0	3	2	14	16,5	
200	243	0	13	8	2	1	16	11	
100	121	6	6	16	1		18	5,5	
90	109	4	10	0					
80	97	2	13	8	15		17	2,1	
70	85	0	16	16	14		15	22,76	
60	72	11	0	0	13		14	19,42	
50	60	9	3	8	12		13	15,68	
40	48	7	6	16	11		12	12,74	
30	36	5	10	0	10		11	9,4	
20	24	3	13	8	9		10	6,66	
10	12	1	16	16	8		9	2,72	
9	10	11	5	0	7		8	23,38	
8	9	8	13	8	6		7	20,04	
7	8	6	1	16	5		6	16,17	
6	7	3	10	0	4		5	13,36	
5	6	0	18	8	3		4	10,02	
4	4	10	6	16	2		3	6,68	
3	3	7	15	0	1		1	3,34	
2	2	5	3	8	$\frac{1}{2}$			20,51	
1	1	2	11	16	$\frac{1}{4}$			13,67	

A P P E N D I X.

CONGRESS have made foreign Gold and Silver Coins a legal tender for the payment of all debts and demands, at the several and respective rates following, viz. The Gold Coins of Great Britain and Portugal, of their present standard, at the rate of 100 cents for every 27 grains of the actual weight thereof—Those of France and Spain, $27\frac{2}{3}$ grains of the actual weight thereof—Spanish milled Dollars, weighing 17 pwt. 7 gr. equal to 100 cents, and in proportion for the parts of a dollar—Crowns of France, weighing 18 pwt. 17 gr. equal to 110 cents, and in proportion for the parts of a Crown. They have enacted, That every cent shall contain 208 grains of copper, and every half cent 104 grains; agreeably to which, the following Tables are calculated.

WEIGHTS of several Pieces of English, Portuguese, and French Gold Coins, according to a late Act of Congress, passed November, 1792.

	Pwt.	Gr.	Doll.	Cents.	Mills.
Double Johannes	18		16	0	0
Single ditto -	9		8		
English Guinea	5	6	4	$66\frac{2}{3}$	
Half ditto -	2	15	2	$33\frac{1}{3}$	
French Guinea	5	6	4	59	8
Half ditto -	2	15	2	29	9
4 Pistoles -	16	12	14	45	2
2 Pistoles -	8	6	7	22	6
1 Pistole -	4	3	3	61	3
Moidore -	6	22	6	14	8

T A B L E S.

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Weight of English and Portuguese Gold, in Dollars,
Cents and Mills, agreeably to an Act of Congress pass-
ed in the Session of November, 1792.

Grs.	Cents.	Mills.	Pwts.	Dols.	Cents.	Mills.
1	3	7	1	0	89	
2	7	4	2	1	77 $\frac{1}{2}$	
3	11	1	3	2	66 $\frac{2}{3}$	
4	14	8	4	3	55 $\frac{1}{2}$	
5	18 $\frac{1}{2}$		5	4	44	4
6	22	2	6	5	33 $\frac{1}{3}$	
7	25	9	7	6	22	2
8	29	6	8	7	11	1
9	33 $\frac{1}{3}$		9	8		
10	37		10	8	89	
11	40	7	11	9	77 $\frac{1}{2}$	
12	44	4	12	10	66 $\frac{2}{3}$	
13	48	1	13	11	55 $\frac{1}{2}$	
14	51	8 $\frac{1}{2}$	14	12	44	4
15	55	5 $\frac{1}{2}$	15	13	33 $\frac{1}{3}$	
16	59 $\frac{1}{4}$		16	14	22	2
17	63		17	15	11	1
18	66 $\frac{2}{3}$		18	16		
19	70	4	19	16	89	
20	74	0 $\frac{1}{2}$	20.			
21	77 $\frac{1}{2}$		1	17	77 $\frac{1}{2}$	
22	81 $\frac{1}{2}$		2	35	55 $\frac{1}{2}$	
23	85	2	3	53	33 $\frac{1}{3}$	

Weight

T A B L E S.

WEIGHT of French and Spanish Gold, in Dollars, Cents and Mills, agreeably to the act aforesaid.

Grs.	Cents.	Mills.	Pwts.	Dolls.	Cents.	Mills.
1	3	6 $\frac{1}{2}$	1	0	87	6
2	7	3	2	1	75	2
3	11		3	2	62 $\frac{1}{4}$	
4	14	6	4	3	50	8 $\frac{1}{2}$
5	18	2 $\frac{1}{2}$	5	4	38	
6	21	9	6	5	25	5 $\frac{1}{2}$
7	25	5 $\frac{1}{2}$	7	6	13	1 $\frac{1}{3}$
8	29	2	8	7	0	7 $\frac{1}{4}$
9	32	8 $\frac{1}{2}$	9	7	88	3
10	36 $\frac{1}{2}$		10	8	76	
11	40	1 $\frac{1}{2}$	11	9	63 $\frac{1}{2}$	
12	43	8	12	10	51	1
13	47	4 $\frac{1}{2}$	13	11	38	7
14	51	1	14	12	26	3
15	54 $\frac{1}{4}$		15	13	13	9
16	58	4	16	14	1 $\frac{1}{2}$	
17	62	0 $\frac{1}{2}$	17	14	89	0 $\frac{1}{2}$
18	65	7	18	15	76	6 $\frac{1}{2}$
19	69	3 $\frac{1}{2}$	19	16	64	2 $\frac{1}{2}$
20	73		oz.			
21	76	6 $\frac{1}{2}$	1	17	51	8 $\frac{1}{2}$
22	80	3	2	35	3	6 $\frac{1}{2}$
23	83	0 $\frac{1}{2}$	3	52	55 $\frac{1}{2}$	



F I N I S.

